PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)
BSc DEGREE EXAMINATION DECEMBER 2019
(Second Semester)

Branch – **MATHEMATICS**

ANALYTICAL GEOMETRY OF 3D AND VECTOR CALCULUS

Time : Three Hours   Maximum : 75 Marks

SECTION-A (20 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks   (10 x 2 = 20)

1   Define Sphere.
2   Find the equation o the sphere with centre (1,2,3) and radius 4.
3   Define Right circular cone.
4   Define guiding curve of a cylinder.
5   Write the condition for the plane lx+my+nz=p to touch the conicoid $ax^2+by^2+cz^2=1$.
6   Define Enveloping cylinder.

7   If $\vec{r}$ is a constant magnitude vector find $\vec{r} \cdot \dfrac{d\vec{r}}{dt}$ & $\vec{r} * \dfrac{d\vec{r}}{dt}$

8   Find the unit tangent vector on the curve $x=t^2+1$, $y=4t-3$, $z=2t^2-6t$ at t=2.
9   Define conservative functions.
10   If $\vec{F} = 3xy i - y^2 j$, find $\int_c \vec{F} \circ d\vec{r}$ where c is the curve on the xy plane $y=2x^2$

     from (0,0) to (1,2).

SECTION - B (25 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 5 = 25)

11 a   Find the equation to the sphere through the four points (0,0,0). (a,0,0), (0,b,0) and (0,0,c).
                              OR
   b   A sphere of constant radius 2k passes through the origin and meets the axes in A,B,C. Find the locus of the centroid of the tetrahedron OABC.

12 a   Find the equations of the cone with vertex O and base curve the conic in which surface $ax^2+by^2+cz^2=1$ is cut by the plane $l_1x+m_1y+n_1z=p$.
                              OR
   b   Find the equation of the tangent planes to the cone $9x^2-4y^2+16z^2=0$ which contain the line $\dfrac{x}{32} = \dfrac{y}{72} = \dfrac{z}{27}$.

13 a   Find the equation of the cylinder whose generators are parallel to the z-axis and the guiding curve is $ax^2+by^2=cz$, lx+my+nz=p.
                              OR
   b   Derive the condition for the plane lx+my+nz=p to touch the conicoid $ax^2+by^2+cz^2=1$.

14 a   Find the angle between the normal to the surface $xy-z^2=0$ at the points (1,4,-2) and (-3,-3,3).
                              OR
   b   If $\vec{F} = xy^2 i + 2x^2 yz j - 3yz^2 k$ find $\text{div}\vec{F}$ and $\text{curl}\vec{F}$ at (1,-1,1).

Cont...

15 a Find the total work done in moving a particle in a force field given by $\vec{F} = 3xy\vec{i} - 5z\vec{p} + 10x\vec{k}$ along $x=t^2+1$, $y=2t^2$ & $z=t^3$ from $t=1$ to $t=2$.

OR

b Find the common area between $y^2=4x$ and $x^2=4y$ by using Green's theorem.

SECTION - C (30 Marks)
Answer any **THREE** Questions
ALL Questions Carry **EQUAL** Marks (3 x 10 = 30)

16 Find the equation of the sphere which passes into the circle $x^2+y^2+z^2-2x-4y=0$, $x+2y+3z=8$ and touches the plane $4x+3y=25$.

17 Find the condition for $ax^2+by^2+cz^2+2fyz+2gzx+2fxy=0$ to represent a right circular cone. Obtain the equation of the axis and the vertical angle of the cone.

18 Find the equation of the right circular cylinder described on the circle through the points $(a,0,0)$, $(0,a,0)$, $(0,0,a)$ as a guiding curve.

19 Find $\nabla\times(\nabla\times\vec{F})$ and hence deduce that $\nabla\times\nabla\times\nabla\times(\nabla\times\vec{F}) = \nabla^4\vec{F}$ if $\vec{F}$ is solenoidal.

20 Verify Gauss theorem for $\vec{F} = 4x\vec{i} - 2y^2\vec{j} + z^2\vec{k}$ taken over the region bounded by $x^2+y^2=4$, $z=0$ and $z=3$.

Z-Z-Z   END