# PSG COLLEGE OF ARTS & SCIENCE
## (AUTONOMOUS)

### BSc DEGREE EXAMINATION DECEMBER 2019
(Second Semester)

Branch – **MATHMATICS**

## ANALYTICAL GEOMETRY

Time: Three Hours $\qquad$ Maximum: 75 Marks

### SECTION-A (10 Marks)
Answer **ALL** questions
**ALL** questions carry **EQUAL** marks $\qquad$ (10 x 1 = 10)

1   What is the directrix corresponding to the pole.

(i) $\dfrac{r}{l} = e\cos(\theta - \alpha)$ $\qquad$ (ii) $\dfrac{1}{r} = e\cos(\theta - \alpha)$

(iii) $\dfrac{1}{r} = e\cos(\alpha - \theta)$ $\qquad$ (iv) $\dfrac{1}{r} = e\cos(\theta - \alpha)$

2   What is the semi-latus rectum of $\dfrac{10}{\gamma} = 3\cos\theta + 4\sin\theta + 5$.

(i)  4 $\qquad$ (ii)  1/2

(iii)  5 $\qquad$ (iv)  2

3   Find the condition that two given straight lines should be co-planar.

(i) $\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l & m & n \\ l_1 & m_1 & n_1 \end{vmatrix} = 0$ (ii) $\begin{vmatrix} x_2 + x_1 & y_2 + y_1 & z_2 + z_1 \\ l & m & n \\ l_1 & m_1 & n_1 \end{vmatrix} = 0$

(iii) $\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l & m & n \\ l_1 & m_1 & n_1 \end{vmatrix} \neq 0$ (iv) $\begin{vmatrix} x_2 + x_1 & y_2 + y_1 & z_2 + z_1 \\ l & m & n \\ l_1 & m_1 & n_1 \end{vmatrix} \neq 0$

4   Which of the following lines are not intersected .
(i)  Parallel $\qquad$ (ii)  Skew
(iii) Coplanar $\qquad$ (iv)  Perpendicular

5   Find the equation of the sphere with centre (-1,2,-3) and radius 3 units.
(i) $x^2+y^2+z^2-4x+8y-12z-10=0$ (ii) $x^2+y^2+z^2-2x+4y-6z-5=0$
(iii) $x^2+y^2+z^2+2x-4y+6z+5=0$ (iv) $x^2+y^2+z^2+4x-8y+12z+10=0$

6   What is the plane section of a sphere?
(i)  Director circle $\qquad$ (ii)  Small circle
(iii) Circle $\qquad$ (iv)  Great circle

7   Identify the equation of the right circular cone with vertex O, axis Z – axis and semi – vertical angle $\alpha$ is
(i) $x^2+y^2=\tan^2\alpha$ $\qquad$ (ii) $x^2+y^2 =z^2 \tan^2\alpha$
(iii) $x^2+y^2=z^2 \tan\alpha$ $\qquad$ (iv) $x^2+y^2=z^2\tan^2\alpha$

8  Identity the condition for the plane lx+my+nz=0 to touch the quadric cone ax$^2$+by$^2$+cz$^2$+2fyz+2gzx+2hxy=0.

(i) $\begin{vmatrix} a & h & g & l \\ h & b & f & m \\ g & f & c & n \\ l & m & n & o \end{vmatrix} = 0$

(ii) $\begin{vmatrix} a & h & g & l \\ h & b & f & m \\ l & m & n & o \\ g & f & c & n \end{vmatrix} = 0$

(iii) $\begin{vmatrix} a & h & g & l \\ g & f & c & n \\ h & b & f & m \\ l & m & n & o \end{vmatrix} = 0$

(iv) $\begin{vmatrix} h & b & f & m \\ a & h & g & l \\ g & f & c & n \\ l & m & n & o \end{vmatrix} = 0$

9  Which is parallel to the generator of the cylinder?
(i) Axes                    (ii) Directrix
(iii) line                  (iv) Latus rectum

10  A cylinder is a surface generated by a _____.
(i) Tangent line            (ii) Straight line
(iii) Normal line           (iv) Initial line

## SECTION - B (35 Marks)
### Answer ALL Questions
### ALL Questions Carry EQUAL Marks (5 x 5 = 25)

11  a  State the condition that the line $\dfrac{1}{r} = A\cos\theta + B\sin\theta$ may be a tangent to the conic $\dfrac{1}{r} = 1 + e\cos\theta$.

OR

b  If the normal at the point P on a conic meets axis in G, then SG=e.sp.

12  a  Show that the image of the point (1,-2,3) in the plane 2x-3y+2z+3=0 is (-3,4,-1).

OR

b  Calculate the shortest distance between the lines
$$\frac{x-3}{-1} = \frac{y-4}{2} = \frac{z+2}{1} ; \frac{x-1}{1} = \frac{y+7}{3} = \frac{z+2}{2} .$$

13  a  Bring out the equation of the sphere through the four points and determining its radius (0,0,0),(9,0,0),(0,b,0),(0,0,C).

OR

b  Bring out the eqn of the sphere which touches the sphere x$^2$+y$^2$+z$^2$-6x+2z+1=0 at the point (2,-2,1) and passes through the origin.

14  a  State the equation of the cone with or origin which passes through the curve is ax$^2$+by$^2$+cz$^2$=1;lx+my+nz=ρ.

OR

b  Find the equation of the right circular cone whose vertex is at the origin, whose axis is the line $\dfrac{x}{1} = \dfrac{y}{2} = \dfrac{z}{3}$ and which has a semi vertical angle of 60$^0$.

15 a Bring out the equation of the right circular cylinder of radius 3 with the axis $\dfrac{x+2}{3} = \dfrac{y-4}{6} = \dfrac{z-1}{2}$.

OR

b Out line about i) Cylinder ii) Right circular cylinder

iii) Enveloping cylinder.

## SECTION -C (40 Marks)
### Answer ALL questions
### ALL questions carry EQUAL Marks          (5 x 8 = 40)

16 a Derive the tangents at the extremities of any focal chord of a conic intersect on the corresponding directrix.

OR

b If a focal chord of an ellipse makes an angle $\alpha$ with the major axis, show that the angle between the tangents at its extremities is $\tan^{-1}\left(\dfrac{2e\sin\alpha}{e^2-1}\right)$

17 a Show that the lines $\dfrac{x-3}{2} = \dfrac{y-2}{-5} = \dfrac{z-1}{3} ; \dfrac{x-1}{-4} = \dfrac{y+2}{1} = \dfrac{z-6}{2}$ are coplanar and find the equation of the plane determined by them.

OR

b Calculate the shortest distance between the lines
$\dfrac{x-3}{-3} = \dfrac{y-8}{1} = \dfrac{z-3}{-1} ; \dfrac{x+3}{3} = \dfrac{y+7}{-2} = \dfrac{z-6}{-4}$ and find the equn of the line of the S.D also.

18 a Enumerate the equation of the sphere through the points (2,3,+1), (5,-1,2), (4,3,-1) and (2,5,3).

OR

b Bring out the equation of the sphere which passes through the circle $x^2+y^2+z^2-2x-4y = 0$ and $x+2y+3z = 8$ and touches the plane $4x+37=25$.

19 a Show that the plane $\dfrac{x}{a}+\dfrac{y}{b}+\dfrac{z}{c}=1$ meets the co-ordinate axis in (A,B,C). Prove that the equation of the cone generated by the lines drawn form O to meet the circle A, B, C is $yz\left(\dfrac{b}{c}+\dfrac{c}{b}\right)+zx\left(\dfrac{a}{c}+\dfrac{c}{a}\right)+xy\left(\dfrac{a}{b}+\dfrac{b}{a}\right)=0$.

OR

b Find the equation of the tangent plane of the cone $9x^2 - 4y^2 +16z^2 = 0$ which contain the line $\dfrac{x}{32} = \dfrac{y}{f2} = \dfrac{z}{2f}$.

20 a Bring out the equation to the cylinder whose generators are parallel to the line $\dfrac{x}{1} = \dfrac{y}{-2} = \dfrac{z}{3}$ and guiding curve $x^2+2y^2=1$, z=3.

OR

b Discuss the equation of the right circular cylinder described on the circle passing through the points (a,0,0),(0,a,0),(0,0,a) as a guiding curve.

Z-Z-Z