14MAU14

PSG COLLEGE OF ARTS & SCIENCE

(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2019

(Fifth Semester)

Branch – **MATHEMATICS**

REAL ANALYSIS

Time : Three Hours Maximum : 75 Marks

SECTION-A (20 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** marks (10 x 2 = 20)

1 Define equivalent equation.

2 Prove that let E be a non-empty set of real numbers which is bounded above.

3 State the Heine-Borel theorem.

4 Prove that if E is an infinite subset of a compact set K, then E has a limit point in K.

5 Prove that the series $\frac{1}{2}+\frac{1}{3}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{2^3}+\frac{1}{3^3}+.....$ is converges.

6 Prove that if $0 \le x < 1$ then $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$ if $x \ge 1$ the series diverges.

7 Write the triangle inequality.

8 Define uniformly continuous.

9 State the mean value theorem.

10 Define L^1 Hospital rule.

SECTION - B (25 Marks)

Answer **ALL** Questions

ALL Questions Carry **EQUAL** Marks (5 x 5 = 25)

11 a Prove that every infinte subset of a countable set is countable.

OR

b Prove that let $\{E_n\}$, n=1,2,3…. Be a sequence of countable sets, and let $S = \bigcup_{n=1}^{\infty} E_n$ then S is countable.

12 a Let p be a non-empty perfect set in R^k then prove that p is uncountable.

OR

b Prove that a subset E of the line R^1 is connected iff it has the property if $x \in E$, $y \in E$, and $x<z<y$, then $z \in E$.

13 a State and prove root test.

OR

b State and prove Ratio-test.

14 a Suppose f is a continuous mapping of a compact metric space X into a metric space Y. Then prove that f(X) is compact.

OR

b Prove that if f is continuous of a metric space X into a metric space Y and if E is connected subset of X then f(E) is connected.

15 a Suppose f is a continuous mapping of [a,b] into R^k and f is differentiable in (a,b). then prove that there exists $x \in (a,b)$ such that $|f(b)-f(a)| \le (b-a)|f'(x)|$.

OR

b State and prove Chain rule for differentiation theorem.

SECTION - C (30 Marks)

Answer any **THREE** Questions

ALL Questions Carry **EQUAL** Marks (3 x 10 = 30)

16 Prove the followings:
If X is a metric space and $E \subset X$ then
a) $\overline{E}$ is closed
b) $E = \overline{E}$ iff E is closed
c) $\overline{E} \subset F$ for every closed set $F \subset X$ such that $E \subset F$.

17 If a set E in R^k has one of the following three properties, then it has the other two
a) E is closed and bounded
b) E is compact
c) every infinite subset of E has a limit point in E.

18 Prove that $\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n = e.$

19 Let f be a continuous mapping of a compact metric space X into a metric space Y then proves that f is uniformly continuous on X.

20 State and prove Taylor's theorem.

Z-Z-Z END