18MAU07

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)
BSc DEGREE EXAMINATION DECEMBER 2019
(Third Semester)

Branch – **MATHEMATICS**

PARTIAL DIFFERENTIAL EQUATONS AND FOURIER TRANSFORM

Time: Three Hours Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 x 1 = 10)

1 The complete solution is
(i) f(x,y,z)=0 (ii) f(x,y,z,a,b)=0 (iii)f(x,y,z)=1 (iv) f(x,y,z,a,b)=1

2 The separable equation is
(i) f(x,p)=F(y,p) (ii) f(x,p)=y (iii) f(x,p)=0 (iv) f(x)=F(y,q)

3 In hyperbolic type of PDE, discriminant part should be
(i) equal to 0 (ii) equal to 1 (iii) les than 0 (iv) greater than 1

4 The roots re equal then the complementary function of f(D,D')z=0 is
(i) φ(y+mx)+φ(y-mx) (ii) $\varphi_1(y+mx)+x\varphi_2(y+mx)$
(iii) φ(y+mx)+φ(y+mx) (iv) φ(y+mx)+xφ(y+mx)

5 The periodic function of cos x is
(i) 2π (ii) π (iii) $\frac{\pi}{2}$ (iv) $\frac{\pi}{4}$

6 The Fourier function f(x) of half range cosine series at (0,π) is
(i) $\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos nx$ (ii) $\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\sin nx$
(iii) $\sum_{n=1}^{\infty}a_n\sin nx$ (iv) $\sum_{n=1}^{\infty}a_n\cos nx$

7 If f(x) is an odd function then the Fourier integral is
(i) $\frac{2}{\pi}\int_0^{\infty}\int_0^{\infty}f(t)\sin st\sin sx\,dt\,ds$ (ii) $\int_0^{\infty}\int_0^{\infty}f(t)\sin st\sin sx\,dt\,ds$
(iii) $\frac{2}{\pi}\int_0^{\infty}\int_0^{\infty}f(t)\sin sx\,ds$ (iv) $\frac{2}{\pi}\int_0^{\infty}\int_0^{\infty}f(t)\sin st\,dt$

8 |x|cos sx, where –a<x<a is an even function in the interval of
(i) (-a,a) (ii) (-a,0) (iii)(0,a) (iv) (0,1)

9 Heat conduction problem is ______.
(i) $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$ (ii) $\frac{\partial u}{\partial t}=-\frac{\partial^2 u}{\partial x^2}$ (iii) $\frac{\partial u}{\partial t}=-k^2\frac{\partial^2 u}{\partial x^2}$ (iv) $\frac{\partial u}{\partial t}=k^2\frac{\partial^2 u}{\partial x^2}$

10 Two dimensional steady state heat flow equation is
(i) $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ (ii) $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$
(iii) $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=1$ (iv) $\frac{\partial^2 u}{\partial x^2}=-\frac{\partial^2 u}{\partial y^2}$

SECTION - B (25 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** Marks (5 x 5 = 25)

11 a Obtain PDE by eliminating the arbitrary function from $z=xy+f(x^2+y^2)$.

12 a Find the characteristic equations of $x^2u_{xx}-2xyu_{xy}+y^2u_{yy}=e^x$.

OR

b Solve $(D^2 - 4DD' + 3D'^2)z = 0$.

13 a Find the characteristic expansion of $f(x)=x$ in $-\pi<x<\pi$.

OR

b Expand the function $f(x)=x$, $0<x<\pi$ in Fourier sine series.

14 a Derive the Fourier integral theorem.

OR

b Find the Fourier transform of $f(x)=1$ in $|x|<a$.

15 a Solve the equation $\frac{\partial u}{\partial t} = k^2 \frac{\partial^2 u}{\partial x^2}$ subject to the condition $u(l,t)=0$.

OR

b Find the steady state temperature distribution in a rectangular plate of sides a and b insulated at the lateral surface and satisfying the boundary conditions $u(0,y)=0$ for $0\le y\le b$.

SECTION -C (40 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 8 = 40)

16 a Find the general equation of the following linear PDE $(y+zx)p-(x+yz)q=x^2-y^2$.

OR

b Find the complete integral of the PDE $p^2z^2+q^2=1$.

17 a Derive the canonical form of $u_{xx}+2u_{xy}+4u_{yy}+2u_x+3u_y=0$.

OR

b Solve $\frac{\partial^2 z}{\partial x^2} - 5\frac{\partial^2 z}{\partial x \partial y} + 6\frac{\partial z^2}{\partial y^2} = e^{x+y}$.

18 a Expand $f(x)=x(2\pi-x)$, as Fourier series in $(0,2\pi)$ and hence deduce that $1+\frac{1}{2^2}+\frac{1}{3^2}+\ldots=\frac{\pi^2}{6}$

OR

b Find the half range sine series for sin ax in $0<x<\pi$.

19 a Find the Fourier cosine integral of the function e^{-ax}. Hence deduce the value of the value $\frac{2}{\pi}\int_0^\infty \frac{\cos\lambda x}{1+\lambda^2}d\lambda$.

OR

b Find the Fourier transform of $\frac{1}{\sqrt{|x|}}$.

20 a Derive the solution of heat equation.

OR

b Derive the two dimensional heat flow equations.

Z-Z-Z END