14MAU07

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2019
(Third Semester)

Branch – **MATHEMATICS**

PARTIAL DIFFERENTIAL EQUATIONS AND FOURIER TRANSFORM

Time : Three Hours Maximum : 75 Marks

SECTION-A (20 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 x 2 = 20)

1 Write the general form of Lagrange's equation.
2 Define singular integral of a partial differential equation.
3 Solve: $z = px + qy + f(p, q)$.
4 Find the general solution of $2p + 3q = 1$.
5 If $f(x) = \frac{1}{2}(\pi - x),\ 0 < x < 2\pi$, find a_n.
6 Define Fourier series expansion of odd function.
7 State the Fourier integral theorem.
8 Define Parseval's identity.
9 Write down the inversion formula for finite Fourier sine transform.
10 Prove that $Fs\{f'(x)\} = \frac{-p\pi}{1}\overline{f_c}(p)$.

SECTION - B (25 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 5 = 25)

11 a Solve: $P(1 + q^2) = q(z-1)$.
OR
b Eliminate the arbitrary function : $ax + by + cz = f(x^2 + y^2 + z^2)$.

12 a Solve : $x^2 \frac{\delta z}{\delta x} + y^2 \frac{\delta z}{\delta y} = (x + y)z$.
OR
b Solve: $yzp + zxq = xy$.

13 a Express $f(x) = x, (-\pi < x < \pi)$ as a Fourier series with period 2π.
OR
b Find the half- range sine series for $f(x) = c$ in the range 0 to π.

14 a If $F\{f(x)\} = F(s)$ then prove that $F\{f(x) \cos 9x\} = \frac{1}{2}\{F(s-a) + F(s+a)\}$.
OR
b Using Parseval's identity, prove that $\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt = \pi/2$.

15 a Find Fourier cosine transform of $f(x) = \begin{cases} \cos x & \text{in } 0 < x < a \\ 0, & x \geq a \end{cases}$.
OR
b Find the finite Fourier sine and cosine transform of $f(x) = 1$ in $(0, \pi)$.

Cont...

SECTION - C (30 Marks)

Answer any **THREE** Questions

ALL Questions Carry **EQUAL** Marks (3 x 10 = 30)

16 Solve: $(x^2 - yz)P + (y^2 - zx)q = z^2 - xy$.

17 Solve: $p^2 + q^2 - 2px - 2qy + 1 = 0$ by Charpit's method.

18 Show that $x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}(-1)^n \frac{\cos nx}{n^2}$ in the internal $(-\pi \leq x \leq \pi)$. Deduce that (i) $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - .... = \frac{\pi^2}{12}$ (ii) $\frac{1}{1^2} - \frac{1}{3^2} + \frac{1}{5^2} - .... = \frac{\pi^2}{8}$.

19 Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2 & \text{if } |x| < 1 \\ 0 & \text{if } |x| > 1 \end{cases}$. Hence evaluate $\int_0^{\infty} \left(\frac{x\cos x - \sin x}{x^3}\right) \cos\frac{x}{2} dx$.

20 Solve $\frac{\delta u}{\delta t} = \frac{\delta^2 u}{\delta x^2}$ for $x > 0, t > 0$ given that, (i) $u(0, t) = 0$ for $t > 0$

(ii) $u(x, 0) = \begin{cases} 1 & \text{for } 0 < x < 1 \\ 0 & \text{for } x \geq 1 \end{cases}$ (iii) $u(x, t)$ is bounded.

Z-Z-Z END