14MAU13

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2019
(Fifth Semester)

Branch – **MATHEMATICS**

ALGEBRA – I

Time : Three Hours — Maximum : 75 Marks

SECTION-A (20 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 x 2 = 20)

1 Define Onto mapping.
2 State Fermat's theorem.
3 Define factor group.
4 Define Kernel of ϕ.
5 Define an automorphism.
6 Define transpiration on permutation group.
7 Define zero divisor.
8 Define skew field.
9 Define principal ideal.
10 Define greatest common divisor.

SECTION - B (25 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 5 = 25)

11 a If H is a non empty finite subset of a group G and H is closed under multiplication, then prove H is a subgroup of G.

OR

b State and prove Euler's theorem.

12 a If H and K are finite subgroups of G of orders O(H) and O(K) respectively, then show $O(HK) = \frac{O(H)O(K)}{O(H \cap K)}$.

OR

b If ϕ is a homomorphism of g into $\overline{G}$ with Kernel K, then prove K is a normal subgroup of G.

13 a Let S be a set and $\theta \in A(S)$. If $a,b \in S$, define $a \equiv_{\theta} b$ if and only if $b = a\theta^i$ for some integer i. show that this defines an equivalence relation on S.

OR

b Prove that every permutation is the product of its cycles.

14 a If p is a prime number then prove J_p, the ring of integers mod p, is a field.

OR

b If R is a ring, then prove for all $a,b \in R$ (i) $a0=0a=0$ (ii) $a(-b)=(-a)b=-(ab)$.

15 a Let R be a commutative ring with unit element whose only ideals are (0) and R itself. Then show that R is a field.

OR

b Let R be a Euclidean ring. If $a,b,c \in R$, $a|bc$ but $(a,b)=1$, then show that $a|c$.

SECTION - C (30 Marks)
Answer any **THREE** Questions
ALL Questions Carry **EQUAL** Marks (3 x 10 = 30)

16 Prove the relation $a \equiv b \bmod H$ is an equivalence relation.

17 State and prove Cauchy's theorem for Abelian groups.

18 State and prove Cayley's theorem.

19 Show that a finite integral domain is a field.

20 Prove the field $A=(a_0)$ is a maximal ideal of the Euclidean ring R if and only if [illegible]