PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION MAY 2018
(First Semester)

Branch – STATISTICS

MEASURE THEORY

Time: Three Hours   Maximum: 75 Marks

Answer **ALL** questions

ALL questions carry **EQUAL** marks   (5 x 15 = 75)

1 a Explain Ring and semi ring.

   b Define sequence of sets with an example.

   OR

   c Prove that Inf g_v = - sup (-g_v).

   d Explain Monotone class and Monotone system.

2 a Prove that continuity theorem.

   b Explain the properties of measure function.

   OR

   c Find the Lebesque – Stieltjes measures of the following sets :
      (i) $\{2\}$,   (ii) [-0.5, 3]   (iii) (-1, 0)$\cup$(1, 2)   (iv) (0.05)$\cup$(1,2),
      (v) $\{x : |x| + 2x^2 > 1\}$.

   d Define Lebesque measure and probability.

3 a Explain Measurable Function.

   b Define convergence in measure with an example.

   OR

   c Find if $\{fn\}$ is a sequence of measurable functions which is fundamental in measure, then there exists a measurable function f such that $f_n \rightarrow f$ in measure.

   d Explain the relationship between convergence in measure and almost everywhere convergence.

4 a Explain integral of a non-negative measurable function.

   b If $f \geq g$ are integrable function such that $f \geq g$ a.e. then prove that $\int fd\mu \geq \int gd\mu$.

   OR

   c Prove that Hahn Decomposition theorem.

   d State and prove that Monotone convergence theorem.

5 a Discuss Absolute continuity of measure.

   b Explain Lebesque Decomposition theorem.

   OR

   c Explain about product space and product measure.

   d State the Fulini's theorem. Give its applications.

Z-Z-Z   END