

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION JUNE 2018
(Third Semester)

Branch - STATISTICS

STATISTICAL INFERENCE -I

Time: 'Three Hours

Maximum: 75 Marks

Answer ALL questions

ALL questions carry EQUAL marks

(5 x 15 = 75)

1 a State the properties of point estimation.

b Describe the following:

(i) Consistency (ii) Sufficient statistics.

OR

c State and prove the Factorization theorem.

d Find joint sufficient statistics for $(a, 3)$ based on a random sample drawn from Gamma $(a, 3)$ distribution. Verify whether they are minimal sufficient also.

2 a State and prove Crammer - Rao inequality.

b If T^* is the minimum variance unbiased estimator and T is any other unbiased estimator of θ , what is the correlation co-efficient between T^* and T ?

OR

c Prove that uniformly minimum variance unbiased estimator is unique,

d State and prove Rao-BlackwellFs theorem.

.3 a Write a short note on modified minimum chi-square estimator.

b Prove that the multinomial context, the minimum chi-square estimate and maximum likelihood estimate are asymptotically the same.

OR

c Let $X \sim N(\mu, \sigma^2)$, $\mu \in \mathbb{R}$ and $\sigma^2 \in (0, \infty)$ and the likelihood function is

$$L(\mu, \sigma^2 | x) = \frac{1}{\sigma \sqrt{2\pi}} \exp\left(-\frac{(x - \mu)^2}{2\sigma^2}\right)$$

f ind (i) Maximum likelihood estimator for μ (σ^2 is known)

(ii) Maximum likelihood estimator of σ^2 (μ is known),

d Describe the following: i) Asymptotic normality (ii) Asymptotic efficiency.

4 a Define exponential family of distribution and also explain location and scale family.

b Describe the location invariant estimator.

OR

c Elucidate the construction of statistical decision problem and taking decisions applying the minimax and Bayes approaches.

5 a Describe the different methods of finding the confidence intervals.

b Construct the $100(1 - \alpha)\%$ shortest length confidence interval for μ based on a random sample drawn from the normal (μ, σ^2) distribution, where σ^2 is unknown.

OR

c Describe the following: (i) Confidence coefficient (ii) Upper confidence limit.