Exam Date & Time: 30-Sep-2020 (02:00 PM - 05:45 PM)

PSG COLLEGE OF ARTS AND SCIENCE

Note: Writing 3hrs: Checking & Inserting Image : 30mins+ Grace Time : 15mins

MSc DEGREE EXAMINATION MAY 2020
(Fourth Semester)

Branch - MATHEMATICS

DISCIPLINE SPECIFIC ELECTIVE - II - COMPUTATIONAL METHODS [18MAP17A]

Marks: 75 **Duration: 225 mins.**

SECTION A

Answer all the questions.

1) The midpoint method is of the form _______. (1)

(i) $y_{i+1} = y_i + hf\left(t_i + \frac{h}{2}, y_i + \frac{h}{2}f(t_i, y_i)\right)$

(ii) $y_{i+1} = y_i + hf(t_i + h, y_i + hf(t_i, y_i))$

(iii) $y_{i+1} = y_i + \frac{h}{2}f\left(t_i + \frac{h}{2}, y_i + \frac{h}{2}f(t_i, y_i)\right)$

(iv) $y_{i+1} = y_i + \frac{h}{2}f(t_i + h, y_i + hf(t_i, y_i))$

2) The difference equation of Adams-Bashforth two-step explicit method is _______. (1)

(i) $y_{i+1} = y_i + \frac{h}{2}[f(t_i, y_i) - 3f(t_{i-1}, y_{i-1})]$

(ii) $y_{i+1} = y_i + \frac{h}{2}[f(t_i, y_i) + 3f(t_{i-1}, y_{i-1})]$

(iii) $y_{i+1} = y_i + \frac{h}{2}[3f(t_i, y_i) + f(t_{i-1}, y_{i-1})]$

(iv) $y_{i+1} = y_i + \frac{h}{2}[3f(t_i, y_i) - f(t_{i-1}, y_{i-1})]$

3) The limit of the sequence $x^{(k)} = \left(1, 2 + \frac{1}{k}, \frac{3}{k^2}, e^{-k}\sin k\right)^t$ with respect to the l_∞ norm is ________. (1)

(i) $x = (1,2,0,1)^t$ (ii) $x = (1,2,3,1)^t$
(iii) $x = (1,2,3,0)^t$ (iv) $x = (1,2,0,0)^t$

4) (1)

The expression of T_j in the matrix form of the Jacobi iteration method is ________.

(i) $T_j = (D-L)^{-1}U$. (ii) $T_j = (D+L)^{-1}U$
(iii) $T_j = D^{-1}(L+U)$ (iv) $T_j = D^{-1}(L-U)$

5) In the following, which one is the nonlinear second – order boundary – value problem? (1)

(i) $y'' = 100y, 0 \le x \le 1, y(0) = 1, y(1) = e^{-10}$

(ii) $y'' = -e^{-2y}, 1 \le x \le 2, y(1) = 0, y(2) = \ln 2$

(iii) $y'' = 100y, 0 \le x \le 1, y(0) = 1, y'(0) = e^{-10}$

(iv) $y'' = -e^{-2y}, 1 \le x \le 2, y(1) = 0, y'(1) = \ln 2$

6) The centered difference formula for $y'(x_i)$ is of the form ________. (1)

(i) $y'(x_i) = \frac{1}{2h}[y(x_{i+1}) - y(x_{i-1})]$ (ii) $y'(x_i) = \frac{1}{h^2}[y(x_{i+1}) - y(x_{i-1})]$

(iii) $y'(x_i) = \frac{1}{2h}[y(x_{i+1}) + y(x_{i-1})]$ (iv) $y'(x_i) = \frac{1}{h}[y(x_{i+1}) - y(x_{i-1})]$

7) An example of the elliptic partial differential equation is ________. (1)

(i) Laplace equation (ii) Poisson equation
(iii) Diffusion equation (iv) wave equation

8) The order of convergence of the Crank- Nicolson method is ________. (1)

(i) $O(k+h)$ (ii) $O(k+h^2)$
(iii) $O(k^2+h)$ (iv) $O(k^2+h^2)$

9) The condition of $\lambda = \alpha k / h$ for which the explicit Finite – Difference method for the wave equation to be stable is (1)

(i) $1 < \lambda < 2$ (ii) $\lambda \le 2$
(iii) $\lambda \le 1$ (iv) $\lambda = 2$

10) Polynomials of linear type in x and y are of the form (1)

(i) $\phi(x, y) = a + bx^2 + cy^2$ (ii) $\phi(x, y) = a + bx^2 + cy^2 + dxy$
(iii) $\phi(x, y) = a + bx + cy$ (iv) $\phi(x, y) = a + bx + cy + dxy$

SECTION B

Answer all the questions.

11)

a) Use the modified Euler method to approximate the solution of $y(2.5)$ to the initial value problem $y' = 1+(t-y)^2, 2 \le t \le 3, y(2) = 1$ with $h = 0.5$. (5)

[OR]

b) Use the Runge- Kutta method of order four with h=0.1 to obtain approximate solution of $y(0.1)$ to the initial value problem $y' = y-t^2+1, 0 \le t \le 1, y(0) = 0.5$ (5)

12) (5)

a) Determine the l^2 norm of $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ -1 & 1 & 2 \end{bmatrix}$

[OR]

b) Find the first two iterations of the Jacobi method for the following linear system using $x^{(0)} = 0$: $10x_1+5x_2=6$, $5x_1+10x_2-4x_3=25$, $-4x_2+8x_3$,$x_4=-11$, $-x_3+5x_4=-11$. (5)

13) a) Show that the boundary value problem $y''+e^{-xy}+\sin y'=0$, for $1 \le x \le 2$, with $y(1)=y(2)=0$, has a unique solution. Also convert the boundary-value problem into the initial-value problems. (5)

[OR]

b) Derive the centered – difference formula for $y''(x_i)$. (5)

14) a) Using Finite – Difference method with h= k=0.25 , write down the form of the linear system of equations for the elliptic partial differential equation. (5)

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0,\ 0 < x < 1,\ 0 < y < 1;$$

$u(x,0) = 0$, $u(x,1) = x$, $0 \le x \le 1$;

$u(0,y) = 0$, $u(1,y) = y$, $0 \le y \le 1$.

[OR]

b) Using the forward – difference method with m = 5 , T=0.02, N=1, approximate the solution to the parabolic partial differential equation. $\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} = 0$, $0 < x < 1, 0 < t$; (5)

$u(0,t)=u(1,t)=0$, $0<t$, $u(x,0) = \sin \pi x$, $0 \le x \le 1$;

15) a) Discuss the procedure of numerical solution of wave equation using finite difference method. (5)

[OR]

b) Obtain a linear polynomial N(x,y) of each vertex of the triangle T with vertices $V_1(1,1)$, $V_2(-1,2)$ and $V_3(0,0)$. (5)

SECTION C

Answer all the questions.

16) a) Consider the initial value problem $y' = y - t^2 + 1, 0 \le t \le 2, y(0) = 0.5$. Use the exact values given from $y(t)= (t+1)^2-0.5e^t$ as starting values and h= 0.2 to compare the approximate solution of y(0.6) from the implicit Adams – Moulton three-step method. (8)

[OR]

b) Apply the Adams fourth-order predictor corrector method with h=0.2 and starting values from the exact solution $y(t) = \frac{1}{5} t\, e^{3t} - \frac{1}{25} e^{3t} + \frac{1}{25} e^{-2t}$ to obtain the approximate solution of y(0.8) to the initial-value problems $y' = t\, e^{3t}-2y, 0 \le t \le 1, y(0) = 0$. (8)

17) a) Find the first four iterations of the Gauss- Seidel techniques for the following linear system using $x^{(0)}=0$: $10x_1-x_2+2x_3=6$, $-x_1+11x_2-x_3+3x_4=25$, $2x_1-x_2+10x_3-x_4=-11$, $3x_2-x_3+8x_4=15$. (8)

[OR]

b) Find the first four iterations of the SOR method with $\omega=1.25$ for the following linear system using $x^{(0)}=1$: $4x_1+3x_2=24$, $3x_1+4x_2-x_3=30$, $-x_2+4x_3=-24$. (8)

18) a) Use the Non linear Shooting Algorithm h= 0.5 to approximate the solution to the boundary value problem $y'' = -(y')^2-y+\ln x$, $1 \le x \le 2, y(1)=0, y(2)=\ln 2$. (8)

[OR]

b) The boundary value problem $y''=4(y-x)$, $0 \le x \le 1, y(0)=0, y(1)=2$ has the solution $y(x)=e^2(e^4-1)^{-1}(e^{2x}-e^{-2x})+x$. use the Linear Finite difference method with h = 0.25 to approximate the solution, and compare the results to the actual solution. (8)

19) a) Use the Poisson finite - difference method with h=k=0.5 to approximate the solution to the elliptic partial differential equation.

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=4,\ 0<x<1,\ 0<y<2;$$

$$u(x,0)=x^2,\ u(x,2)=(x-2)^2,\ 0 \le x \le 1;$$

$$u(0,y)=y^2,\ u(1,y)=(y-1)^2,\ 0 \le y \le 2.$$

(8)

[OR]

b) Using the backward – Difference method with m=4, T= 0.1 and N=2, approximate the solution to the partial differential equation

$$\frac{\partial u}{\partial t}-\frac{\partial^2 u}{\partial x^2}=0,\ 0<x<2,\ 0<t;$$

$$u(0,t)=u(2,t)=0,\quad 0<t,\quad u(x,0)=\sin\frac{\pi}{2}x,\ 0 \le x \le 2.$$

(8)

20) a) Using the explicit Finite – Difference method with m=4, N=4 and T= 1.0, approximate the solution to the wave equation $\frac{\partial^2 u}{\partial t^2}-\frac{\partial^2 u}{\partial x^2}=0,\ 0<x<1,\ 0<t;$

$$u(0,t)=u(1,t)=0,\quad 0<t,\quad u(x,0)=\sin \pi x,\quad 0 \le x \le 1;$$

$$\frac{\partial u}{\partial t}(x,0)=0,\ 0 \le x \le 1.$$

(8)

[OR]

b) (8)

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Use the Finite – Element method to approximate the solution to the partial differential equation.

$$\frac{\partial}{\partial x}\left(y^2 \frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(y^2 \frac{\partial u}{\partial y}\right)-yu=-x, (x,y)\in D,$$

$u(x,0.5)=2x,\ 0\le x\le 0.5,\quad u(0,y)=0,\quad 0.5\le y\le 1,$

$$y^2\frac{\partial u}{\partial x}\cos\theta_1+y^2\frac{\partial u}{\partial y}\cos\theta_2=\frac{\sqrt{2}}{2}(y-x) \text{ for}(x,y)\in S_2.$$

Let M= 2; T_1 have vertices(0,0.5), (0.25,0.75), (0,1); and T_2 have vertices (0,0.5), (0.5,0.5) and (0.25, 0.75)

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