Exam Date & Time: 28-Sep-2020 (10:00 AM - 01:45 PM)

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PSG COLLEGE OF ARTS AND SCIENCE

Note: Writing 3hrs: Checking & Inserting Image : 30mins

BSc DEGREE EXAMINATION MAY 2020
(Sixth Semester)

Branch - MATHEMATICS WITH COMPUTER APPLICATIONS
COMPLEX ANALYSIS [14MCU25]

Marks: 75

Duration: 210 mins.

SECTION A

Answer all the questions.

1) Define analytic in the domain. (2)

2) Write down the Laplace's differential equation. (2)

3) Define ordinary points. (2)

4) What is meant by a Translation? (2)

5) Define a partition of the interval [a,b]. (2)

6) State Gauss mean value theorem. (2)

7) Define a zero of order m. (2)

8) Define a meromorphic function. (2)

9) Write down the formula for residue of $\frac{\phi(z)}{(z-a)^m}$ at z=a. (2)

10) State Jordan's Inequality. (2)

SECTION B

Answer all the questions.

11) State and prove Cauchy-Riemann partial differential equations.

a) (5)

[OR]

b) (5)

Show that the function $e^x(\cos y+i \sin y)$ is holomorphic and find its derivative.

12) a) Consider the transformation $w = \sqrt{(x^2+y^2)}-iy$ and determine the region D^1 of the w-plane corresponding to the region D of the z-plane given by circular disc $x^2+y^2 \leq 1$. (5)

[OR]

b) Let the rectangular region D in the z-plane be bounded by $x=0, y=0, x=2, y=3$. Determine the region D^1 of the w-plane into which D is mapped under the transformation $w = \sqrt{2}\, e^{i\pi/4} z$. (5)

13) a) Using the definitions of an integral as the limit of a sum evaluate the integral $\int_L z\,dz$ where L is any rectifiable are joining the points $z=\alpha$ and $z=\beta$. (5)

[OR]

b) Let f(z) be analytic within and on the boundary C of a simply connected region D and let z_0 be any point within C, then prove that $f^1(z) = \frac{1}{2\pi i} \int_C \frac{f(z)}{(z-z_0)^2} dz$. (5)

14) a) State and prove Liouville's theorem. (5)

[OR]

b) Represent the function $f(z) = \frac{z}{(z-1)(z-3)}$ by a series of negative powers of (z-1) which converges to f(z) when $0<|z-1|<2$. (5)

15) a) If a>0, prove that $\int_0^\infty \frac{dx}{(x^2+a^2)^2} = \frac{\pi}{4a^3}$. (5)

[OR]

b) Apply Calculus of residues to prove that $\int_0^{2\pi} \frac{\cos 2\theta}{5+4\cos\theta} d\theta = \frac{\pi}{6}$. (5)

SECTION C

Answer 3 out of 5 questions.

16) If $u-v=(x-y)(x^2+4xy+y^2)$ and $f(z)=u+iv$ is an analytic function of $z=x+iy$, find f(z) in terms of z. (10)

17) If w=f(z) represents a conformal transformation of a domain D in the z-plane into a domain D of the w-plane then prove that f(z) is an analytic function of z in D. (10)

18) (10)

State and prove Cauchy's Integral formula theorem.

19) State and prove Laurent's theorem. (10)

20) Prove that $\int_0^{\pi} \frac{a\,d\theta}{a^2+\sin^2\theta} = \frac{\pi}{\sqrt{1+a^2}}$, (a>0). (10)

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