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Exam Date & Time: 30-Sep-2020 (10:00 AM - 01:45 PM)

PSG COLLEGE OF ARTS AND SCIENCE

Note: Writing 3hrs: Checking & Inserting Image : 30mins + Grace Time : 15mins

BSc DEGREE EXAMINATION MAY 2020
(Sixth Semester)

Branch - MATHEMATICS
GRAPH THEORY [14MAU22]

Marks: 75 **Duration: 225 mins.**

SECTION A

Answer all the questions.

1) Define connected graph. (2)

2) Define null graph and pendent vertex. (2)

3) Prove that if in a graph G there is one and only one path between every pair of vertices, G is tree. (2)

4) What do you mean by rooted tree? (2)

5) Write any two properties common to the two graphs of Kuratowski. (2)

6) Define embedding. (2)

7) Write the properties of a sub matrix. (2)

8) Define adjacency matrices. (2)

9) What do you mean by balanced graph? (2)

10) Define complete symmetric graph. (2)

SECTION B

Answer all the questions.

11)

a) Prove that the number of vertices of odd degree in a graph is always even. (5)

[OR]

b) Prove that a simple graph with n vertices and k components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges. (5)

12) (5)

a) Prove that a graph G with n vertices, n-1 edges and no circuits is connected.

[OR]

b) Prove that every tree has either one or two centres. (5)

13) a) Write the simplifying steps in elementary reduction. (5)

[OR]

b) Prove that non-planarity of Kuratowski's second graph cannot be planar. (5)

14) a) If B is a circuit matrix of a connected graph G with e edges and n vertices prove that Rank of B = e – n + 1. (5)

[OR]

b) Write any five observations about adjacency matrix. (5)

15) a) Define the following terms:
(i) Equivalence relation (ii) Tournament (iii) isolated vertex. (5)

[OR]

b) Prove that the (i,j) th entry in X' equals the number of different directed edge sequences of r edges from the i^{th} vertex to the j^{th} vertex. (5)

SECTION C

Answer 3 out of 5 questions.

16) Prove that a connected graph G is an Euler graph iff all vertices of G are of even degree. (10)

17) Prove that a tree with n vertices has (n-1) edges. (10)

18) State and prove Euler's formula. (10)

19) If A(G) is an incidence matrix of a connected graph G with n vertices, then prove that the rank of A(G) is n-1. (10)

20) Prove that the determinant of every square sub matrix of A, the incidence matrix of a digraph is 1,-1 or 0. (10)

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