Exam Date & Time: 28-Sep-2020 (10:00 AM - 01:45 PM)

PSG COLLEGE OF ARTS AND SCIENCE

Note: Writing 3hrs: Checking & Inserting Image : 30mins

BSc DEGREE EXAMINATION MAY 2020
(Sixth Semester)

Branch - MATHEMATICS
COMPLEX ANALYSIS [14MAU20]

Marks: 75 **Duration: 210 mins.**

SECTION A

Answer all the questions.

1) Discuss the analyticity of the function f(z)=xy+iy. (2)

2) Write the polar form of Cauchy Riemann equations. (2)

3) Define Jacobian of transformation u=u(x,y), v=v(x,y). (2)

4) What is the necessary and sufficient condition for the transformation w=f(z) to be conformal? (2)

5) State Cauchy's fundamental theorem. (2)

6) Evaluate $\int_c \frac{dz}{z-a}$ where c is any simpe closed curve and z=a is inside c. (2)

7) State Lioville's therem. (2)

8) Find zero's and poles of $\left(\frac{z+1}{z^2+1}\right)^2$. (2)

9) Find the residue at z=0 of the functions $f(z) = e^{1/z}$. (2)

10) Write the Jordan's inequality. (2)

SECTION B

Answer all the questions.

11)

a) If f(z)=u+iv is analytic in a domain D, then prove that u, v satisfy the equations $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$, $\frac{\partial u}{\partial y} = \frac{-\partial v}{\partial x}$ provided the four partial derivatives u_x, u_y, v_x, v_y exist. (5)

[OR] b) If $u=(x-1)^3-3xy^2+3y^2$, determine v so that u+iv is an analytic function of x+iy. (5)

12) a) If α and β are inverse points of a circle, then show that the equation of the circle can be written as $\left|\frac{z-\alpha}{z-\beta}\right| = k$, $k \neq 1$ and k is a real constant. (5)

[OR] b) Let a rectangular domain R be bounded by x=0, y=0, x=2, y=1. Determine the region R^1 of w-plane into which R is mapped under the transformation. (5)

13) a) Evaluate $\int_c |z|\,dz$, where c is upper half part of circle |z|=1. (5)

[OR] b) Using Cauchy's integral formula, calculate $\int_c \frac{z\,dz}{(9-z)^2(z+i)}$ where c is the circle |z|=2 described in positive sense. (5)

14) a) State and prove Cauchy's inequality. (5)

[OR] b) Prove that a function has no singularity in the finite part of the planes or at infinity is constant. (5)

15) a) Evaluate the residues of $\frac{z^2}{(z-1)(z-2)(z-3)}$ at 1,2,3 and infinity and show that their sum is zero. (5)

[OR] b) Evaluate by method of calculus of residues $\int_c \frac{dz}{(z^2+1)(z-4)}$, where c is a circle |z|=3. (5)

SECTION C

Answer 3 out of 5 questions.

16) If $u = \frac{\sin 2x}{\cosh 2y + \cos 2x}$, find the corresponding analytic function f(z)=u+iv. (10)

17) Consider the map $w = \frac{1}{z}$ and determine the region R^1 in w-plane of the infinite strip R bounded by $\frac{1}{4} < y < \frac{1}{2}$. (10)

18) State and prove Cauchy's integral formula. (10)

19) State and prove Laurent's theorem. (10)

20) (10)

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Evaluate by contour integration $\int_0^{2\pi} \frac{d\theta}{a + b\cos\theta}$

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