Exam Date & Time: 26-Sep-2020 (10:00 AM - 01:30 PM)

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PSG COLLEGE OF ARTS AND SCIENCE

Note: Writing 3hrs: Checking & Inserting Image : 30mins
BSc DEGREE EXAMINATION MAY 2020
(Sixth Semester)

Branch - MATHEMATICS
ALGEBRA-II [14MAU19]

Marks: 75 **Duration: 210 mins.**

SECTION A

Answer all the questions.

1) Let $A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{pmatrix}$. Then find AB. (2)

2) Prove $(iA)^* = -iA^*$. (2)

3) Define Vector Space. (2)

4) Define Linear Combination. (2)

5) What is annihilator of vector space? . (2)

6) Prove that $(\mu, \alpha v + \beta\omega) = \bar{\alpha}(\mu, v) + \bar{\beta}(\mu, \omega)$. (2)

7) What are the conditions for the row-reduced matrix? (2)

8) State Cayley-Hamilton theorem. (2)

9) If v is finite-dimensional over F then for S, $T \in A(v)$.
Then prove that $r(ST) \leq r(T)$. (2)

10) Define characteristics root of T. (2)

SECTION B

Answer all the questions.

11)

a) If the matrix products AB and BC are defined, then prove that (AB)C=A(BC). (5)

[OR]

b) Prove that the product of two symmetric matrices is symmetric iff the matrices commute. (5)

12)

a) If V is the internal direct sum of $U_1, U_2, \ldots U_n$, then prove that V is isomorphic to the external direct sum of $U_1, U_2, \ldots U_n$. (5)

[OR]

b) If V is the finite-dimensional vector space, then prove that it contains a finite set $v_1, v_2, \ldots v_n$ of linearly independent elements whose linear span is V. (5)

13)

a) If V and W are of dimensions m and n, respectively, over F, then prove that Hom (V,W) is of dimension mn over F. (5)

[OR]

b) Let V be a finite-dimensional inner product space, then prove that V has an orthonormal set as a basis. (5)

14)

a) Prove that every matrix is row matrix equivalent to a (unique) row-reduced echelon matrix. (5)

[OR]

b) Prove that the characteristics roots of a hermitian matrix are all real. (5)

15)

a) If $\lambda \in F$ is a characteristics root of $T \in A(V)$, then prove that λ is a root of the minimal polynomial of T. In particular, T only has a finite number of characteristic roots in F. (5)

[OR]

b) The set of all nxn matrices over F form an associative algebra, F_n, over F. If V is an n-dimensional vector space over F, then prove that A(V) and F_n are isomorphic as algebras over F. (5)

SECTION C

Answer 3 out of 5 questions.

16)

Show that the given matrix $\frac{1}{5}\begin{pmatrix} -1+2i & -4-2i \\ 2-4i & -2-i \end{pmatrix}$ is unitary. (10)

17)

If V is finite-dimensional and if W is a subspace of V, then prove that W is finite-dimensional, $\dim W \leq \dim V$ and $\dim V/W = \dim V - \dim W$. (10)

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18) State and prove Schwarz is inequality. (10)

19) Verify the Cayley-Hamilton theorem for the matrix, $A=\begin{pmatrix} 1 & -1 & 2 \\ -2 & 1 & 3 \\ 3 & 2 & -3 \end{pmatrix}$. (10)

Hence compute A^{-1}.

20) If A is an algebra, with unit element, over F, then prove that A is isomorphic to a subalgebra of A(V) for some vector space V over F. (10)

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