19MCU05

PSG COLLEGE OF ARTS & SCIENCE

(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2022

(Second Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

ANALITICAL GEOMETRY OF 3D AND VECTOR CALCULUS

Time: Three Hours Maximum: 50 Marks

SECTION-A (5 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** marks (5 x 1 = 5)

1 The equation of the line joining the points (1,-1,2) and (4,2,3) is given by

(i) $\frac{x-1}{3}=\frac{y-1}{3}=\frac{z-2}{1}$ (ii) $\frac{x-1}{3}=\frac{y+1}{3}=\frac{z-2}{1}$

(iii) $\frac{x-1}{3}=\frac{y-1}{3}=\frac{z+2}{1}$ (iv) $\frac{x+1}{3}=\frac{y+1}{3}=\frac{z+2}{1}$

2 The radius of the sphere $x^2+y^2+z^2+2x-4y-6z+5=0$ is

(i) 6 (ii) 3 (iii) 4 (iv) 5

3 The equation to the cone with vertex at the origin and whose geneators pass through the curve $ax^2+by^2=cz$; lx+my+nz=p is given by

(i) $p(ax^2+by^2)=c(nz^2-lzx-myz)$ (ii) $p(ax^2-by^2)=c(nz^2-lzx-myz)$

(iii) $p(ax^2+by^2)=c(nz^2+lzx+myz)$ (iv) $p(ax^2-by^2)=c(nz^2+lzx+myz)$

4 If $\vec{F}=x^2\vec{i}+xy\vec{j}$, then the value of $\int \vec{F}.\vec{dr}$ from (0,0) to (1,1) along the line $y=x$ is given by

(i) $\frac{2}{3}$ (ii) $\frac{3}{2}$ (iii) $\frac{4}{3}$ (iv) $\frac{3}{4}$

5 If $\vec{r}=x\vec{i}+y\vec{j}+z\vec{k}$, then the value of $div\ \vec{r}$ is

(i) 3 (ii) 0 (iii) 2 (iv) 1

SECTION - B (15 Marks)

Answer **ALL** Questions

ALL Questions Carry **EQUAL** Marks (5 x 3 = 15)

6 a Find the equation of the plane through the intersection of the planes $3x-y+2z-4=0$ and x+y+z-2=0 and passing through the point (2,2,1).

(OR)

b Find the angle between the line $\frac{x+1}{2}=\frac{y}{3}=\frac{z-3}{6}$ and the plane $3x+y+z=7$.

7 a Find the equation of the sphere which has its centre at the point (-1,2,3) and touches the plane $2x-y+2z=6$.

(OR)

b Find the equation of the sphere which touches the sphere $x^2+y^2+z^2+2x-6y+1=0$ at the point (1,2,-2) and passes through the origin.

8 a Derive the equation of the right circular cone whose vetex is at (0,0,1), whose axis is the z-axis and which has a semi-vertical angle of 30°.

(OR)

Cont...

b If $x=\frac{y}{2}=\frac{z}{3}$ represent one of a set of three mutually perpenticular generators of the cone $5yz-8zx-3xy=0$, find the equation of the other two.

9 a If $\vec{F}=(3x^2+6y)\vec{i}-14yz\vec{j}+20xz^2\vec{k}$, evaluate $\int_C \vec{F}.\vec{dr}$, where C is the straight line joining (0,0,0) to (1,1,1).

(OR)

b Find the divergence and curl of the vector point function $xy^2\vec{i}+2x^2yz\vec{j}-3yz^2\vec{k}$,

10 a Find the area of the ellipse $x=a\cos\theta, y=b\sin\theta$.

(OR)

b Using Stokes theorem, prove that $\int_c \phi\Delta\psi\vec{dr}=\int_c \psi\Delta\phi\vec{dr}$

SECTION -C (30 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** Marks (5 x 6 = 30)

11 a Prove that the equation $2x^2-6y^2-12z^2+2zx+18yz+xy=0$ represents a pair of planes and find the angle between them.

(OR)

b A line with direction cosines proportional to 2,7,-5 is drawn to intersect the lines $\frac{x-5}{3}=\frac{y-7}{-1}=\frac{z+2}{1}; \frac{x+3}{-3}=\frac{y-3}{2}=\frac{z-6}{4}$. Find the coordinates of the points of intersection and the length intercepted on it.

12 a Find equation of a sphere having its centre on the plane $4x-5y-z=3$ and passing through the circle $x^2+y^2+z^2-2x-3y+4z+8=0; x-2y+z=8$.

(OR)

b Find the equation of the sphere which touches the plane $3x+2y-z+2=0$ at the point P(1,-2,1) and also cuts orthogonally the sphere $x^2+y^2+z^2-4x+6y+4=0$

13 a Find the equation of the cone circumscribing the sphere $x^2+y^2+z^2-4x+6y+4=0$ and having its vertex at (1,1,1).

(OR)

b Find the equation of the tangent planes to the cone $9x^2-4y^2+16z^2=0$ which contains the line $\frac{x}{32}=\frac{y}{72}=\frac{z}{27}$.

14 a Evaluate by green's theorem $\int_C e^{-x}(\sin ydx+\cos ydy)$.

(OR)

b Find the value of the constants a,b,c so that the vector $\vec{F}=(x+2y+az)\vec{i}+(bx-3y-z)\vec{j}+(4x+cy+2z)\vec{k}$ is irrotational.

15 a Using Stoke's theorem evaluate $\int_C[(2x-y)dx-yz^2dy-y^2zdz]$. where C is the circle $x^2+y^2=1$, corresponding to the surface of the sphee of unit radius.

(OR)

b Use divergence theorem to evaluate $\iint_S \vec{F}\hat{n}\,ds$ where $\vec{F}=x^3\vec{i}+y^3\vec{j}+z^3\vec{k}$, and S is the surface of the sphere $x^2+y^2+z^2=a^2$.

Z-Z-Z END