19STU04

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2022
(Second Semester)

Branch – **STATISTICS**

PROBABILITY AND DISTRIBUTIONS - I

Time: Three Hours Maximum: 50 Marks

SECTION-A (5 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 1 = 5)

1. The outcome of a random experiment is called as ________.
   (i) Sample space (ii) Trial (iii) outcome (iv) Event
2. In the case of continuous random variable the probability at a point is always ______.
   (i) negative (ii) zero (iii) positive (iv) finite
3. If the values taken by a random variable are negative, the negative values will have_____.
   (i) Positive probability
   (ii) negative probability
   (iii) May have negative or positive probabilities
   (iv) Insufficient data
4. The first central moment is always______.
   (i) negative (ii) zero (iii) positive (iv) finite
5. If Var(x)=1 , then Var(2x ± 3) =_______.
   (i) 5 (ii) 16 (iii) 27 (iv) 4

SECTION - B (15 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 3 = 15)

6 a State and prove Baye's theorem.

OR

b Let A and B are two events such that P(A)=3/4 and P(B)=5/8 ,show that (a) P(A∪B)≥3/4 (b) 3/8 ≤ P(A∩B) ≤ 5/8.

7 a If $p(x) = \begin{cases} \frac{x}{15}; x=1,2,3,4,5 \\ 0, \ elsewhere \end{cases}$.

find (a) $P\{x = 1 \ or \ 2\}$, and (b) $p\left\{\frac{1}{2} < X < \frac{5}{2} \middle| X > 1\right\}$.

OR

b State and prove additive theorem of expectation.

Cont...

8 a Give the properties of joint distribution function.

OR

b The joint probability density function of a two -dimensional random variable (X,Y) is given by : $f(x,y) = \begin{cases} 2; 0<x<1, 0<y<x; \\ 0, \ elsewhere \end{cases}$.

(i) Find the marginal density function of X and Y.
(ii) Find the conditional density function of Y given X= x and conditional density function of X given Y= y.
(iii) Check for independence of X and Y.

9 a What are the properties of cumulants?

OR

b State and prove weak law of large numbers.

10 a If X and Y are independent continuous r.v.'s then prove that the p.d.f. of U=X+Y is given by:

$h(u) = \int_{-\infty}^{\infty} f_X(v) f_Y(u-v) dv$.

OR

b Let $f(x,y) = 8xy, 0 < x < y < 1$; $f(x,y) = 0$ $elsewhere$. Find

(a) $E(Y|X = x)$, (b) $E(XY|X = x)$, (c) $Var(Y|X = x)$.

SECTION -C (30 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 6 = 30)

11 a The odds that a book on statistics will be favourabl`y reviewed by 3 independent critics are 3 to 2, 4 to 3 and 2 to 3 respectively. What is the probability that of the three reviews :
(i) All will be favourable,
(ii) Majority of the reviews will be favourable,
(iii) Exactly one review will be favourable, and
(iv) At least one of the reviews will be favourable.

OR

b (i) Prove the additive theorem of probability for independent events.
(ii) Prove the multiplicative theorem of probability for independent events.

12 a The diameter, say X, of an electric cable, is assumed to be a continuous random variable with p.d.f. : $f(x) = 6x(1-x), 0 \leq x \leq 1$.
(i) Check that the above is a p.d.f.,
(ii) Obtain an expression for the c.d.f. of X.,
(iii) Compute $P\left(X \leq \frac{1}{2} \middle| \frac{1}{3} \leq X \leq \frac{2}{3}\right)$.

OR

b Suppose that the time in minutes that a person has to wait at a certain bus stop for a bus is found to be a random phenomenon, with a probability function specified by the distribution function is given by

$$F(x) = \begin{cases} 0 \;, & x < 0 \\ \frac{x}{8} \;, 0 \le x < 2 \\ \frac{x^2}{16}, 2 \le x < 4 \\ 1 \;, 1, x \ge 4 \end{cases}$$

(i) Is the distribution function continuous? If so, give the formula for its probability density function.
(ii) What is the probability that a person will have to wait in a bus stop.
(a) more than 2 minutes, (b) less than 2 minutes, and (c) between 1 and 2 minutes?
(iii) what is the conditional probability that the person will have to wait for a bus for
(a) more than 2 minutes, given that it is more than 1 minute
(b) less than 2 minutes , given that it is more than1 minute ?

13 a For the adjoining bivariate probability distribution of X and Y, find:
(i) P(X ≤ 1, Y=2),
(ii) P(X≤ 1),
(iii) P(Y≤ 3), and
(iv) P(X< 3, Y≤ 4).

X \ Y	1	2	3	4	5	6
0	0	0	1/32	2/32	2/32	3/32
1	1/16	1/16	1/8	1/8	1/8	1/8
2	1/32	1/32	1/64	1/64	0	2/64

OR

b Let X and Y be jointly distributed with p.d.f.:

$$f_{XY}(x,y) = \begin{cases} \frac{1}{4}(1+xy), & |x| < 1, |y| < 1 \\ 0 & ,otherwise \end{cases}$$

show that X and Y are not independent but X^2 and Y^2 are independent.

14 a State and prove Chebychev's inequality.

OR

b Let $X_1, X_2, \ldots, X_n$ be jointly normal with $E(X_i) = 0$ and $E(X_i^2) = 1$for all i and $Cov(X_i, X_j) = \rho$ if $|j-i| = 1$ and = 0, otherwise .Examine if WLLN holds for the sequence $|X_n|$.

15 a Two random variable X and Y have the following joint probability density function:

$$f(x,y) = \begin{cases} 2-x-y; 0 \le x \le 1, 0 \le y \le 1 \\ 0, otherwise \end{cases}$$

Find (i) Marginal probability density function of X and Y ;
(ii) Conditional density functions,
(iii) Var(X) and Var(Y) ; and
(iv) Covariance between X and Y.

OR

b Given the joint density function of X and Y as :

$$f(x,y) = \begin{cases} \frac{1}{2}xe^{-y}; 0 < x < 2, y > 0 \\ 0 \;\; ,elsewhere \end{cases}$$

find the distribution of X+Y.

Z-Z-Z END