PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2022
(Sixth Semester)

Branch – **MATHEMATICS**

LINEAR ALGEBRA

Time: Three Hours Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 x 1 = 10)

1 A is orthogonal if and only if ____
(i) $A = \overline{A}^T$ (ii) $A = -\overline{A}^T$
(iii) $A^{-1} = A^T$ (iv) $A^{-1} = -A^T$

2 Any unitary matrix over R is an _____ matrix.
(i) Hermitian (ii) Symmetric
(iii) Orthogonal (iv) Skew symmetric

3 The intersection of two subspaces of a vector space _____
(i) is not a subspace (ii) is a subspace
(iii) need not be a subspace (iv) none of the above

4 A finite dimensional vector space has a basis consisting of a _____ number of vectors.
(i) infinite (ii) no
(iii) finite (iv) equal

5 If V is a vector space over F, then its dual space is ____
(i) V (ii) Hom (V, F)
(iii) F (iv) none of these

6 S is said to be an ____ set if S is orthogonal and $\|x\| = 1$ for all $x \in S$.
(i) normal (ii) product
(iii) regular (iv) orthonormal

7 Zero is a characteristic root if and only if A is a _____ matrix.
(i) non singular (ii) singular
(iii) regular (iv) irregular

8 The characteristic roots of a real symmetric matrix are _____
(i) real (ii) imaginary
(iii) complex (iv) not defined

9 If V is finite dimensional over F, then $T \in A(V)$ is regular iff T maps V onto ____
(i) F (ii) A
(iii) T (iv) V

10 If $T \in A(V)$ and if $\dim_F V = n$, then T can have ____ n distinct characteristic roots in F
(i) atleast (ii) atmost
(iii) no (iv) greater than

Cont...

SECTION - B (25 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** Marks (5 x 5 = 25)

11 a Prove that a square matrix A is (i) symmetric iff $A = A^T$ (ii) skew symmetric iff $A = -A^T$.

OR

b Let A and B be orthogonal matrices of the same order. Then show that (i) A^T is orthogonal (ii) AB is orthogonal.

12 a Let V be a vector space over F. Then prove that a non-empty subset W of V is a subspace of V iff W is closed with respect to vector addition and scalar multiplication in V.

OR

b Show that any subset of a linearly independent set is linearly independent.

13 a If W is a subspace of a vector space V, then prove that A(A(W)) = W.

OR

b If V is a finite dimensional inner product space and W is a subspace of V, then prove that $V = W \oplus W^{\perp}$.

14 a Explain that the characteristic roots of a unitary matrix are all the unit modulus.

OR

b Verify Cayley Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$

15 a If V is finite dimensional over F, and if $T \in A(V)$ is right invertible, then prove that T is invertible. Also if T is invertible, then show that T^{-1} is a polynomial expression in T over F.

OR

b If $\lambda \in F$ is a characteristic root of $T \in A(V)$, then for any polynomial $q(x) \in F[x]$, $q(\lambda)$ is a characteristic root of $q(T)$.

SECTION -C (40 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** Marks (5 x 8 = 40)

16 a Let A be an mxn matrix, B an nxp matrix and C a p x q matrix. Then prove that A(BC) = (AB)C

OR

b Let A and B be symmetric matrices of order n. Then show that (i) A + B is symmetric (ii) AB is symmetric iff AB = BA (iii) AB + BA is symmetric (iv) If A is symmetric, then kA is symmetric where $k \in F$

17 a If V is a vector space over F, W is a subspace of V. Let $V / W = \{W + v / v \in V\}$. Then show that V / W is a vector space over F under the following conditions. (i) $(W + v_1) + (W + v_2) = W + V_1 + v_2$ (ii) $\alpha(W + v_1) = W + \alpha v_1$

OR

b Let V be a vector space over F. Let $S = \{v_1, v_2, \ldots, v_n\}$ and L(S) = W. Then prove that there exists a linearly independent subset S' of S such that $L(S') = W$.

18 a Prove that every finite dimensional inner product space has an orthonormal basis.

OR

b If u, v $\in$ V, then show that $|(u, v)| \leq \|u\|\|v\|$.

19 a If λ is a characteristic root of A, then prove that $f(\lambda)$ is a characteristic root of the matrix $f(A)$ where $f(x)$ is any polynomial.

OR

b State and Prove Cayley Hamilton theorem.

20 a If V is n-dimensional over F and if T $\in$ A(V) has the matrix $m_1(T)$ in the basis $v_1, \ldots, v_n$ and the matrix $m_2(T)$ in the basis $w_1, \ldots, w_n$ of V over F, then establish that there is an element $C \in F_n$ such that $m_2(T) = C\, m_1(T)\, C^{-1}$. In fact, if S is the linear transformation of V defined by $v_iS = w_i$ for I = 1,2,…,n, then C can be chosen to be $m_1(S)$.

OR

b If $\lambda \in F$ is a characteristic root of T $\in$ A(V), then prove that λ is a root of the minimal polynomial of T. In particular, T only has a finite number of characteristic roots in F.

Z-Z-Z END