20MAU22/18MAU21

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)
BSc DEGREE EXAMINATION MAY 2022
(Sixth Semester)
Branch – **MATHEMATICS**

GRAPH THEORY

Time: Three Hours Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 x 1 = 10)

1. If a graph G has more than one component then G is called ____ graph.
(i) Connected (ii) disconnected
(iii) complete (iv) complete bipartite

2 The sum of the degrees of the points of a graph G is_____
(i)Halves the number of lines (ii) same as the number of lines
(iii) Twice the number of (iv) thrice the number of lines

3 Every non trivial tree has atleast two vertices of degree_____
(i)1 (ii) 0 (iii) same (iv) different

4. A graph G with n vertices is called a tree if____
(i) G is disconnected (ii) G is maximally connected
(iii) G is connected and has n-1 edges (iv) G is connected and has n edges

5. Which of the following statement is wrong?
(i)Every planer graph is a plane
(ii)Every planer graph is isomorphic to a plane graph
(iii) K_5 is non-planer (iv) $K_{3,3}$ is non-planer

6 Every planer graph has genus _____
(i)1 (ii) 0 (iii) 2 (iv) 3

7. The number of edges common to a cut set and a circuit is ____
(i) Different (ii) same (iii) even (iv) odd

8. If two graphs have the same circuit matrix then they have ____graph
(i)2-isomorphic (ii) disconnected (iii) isolated (iv) complete

9 _____ in which the in-degree and the out degree are both equal to zero
(i)Isolated vertex (ii) initial vertex (iii) end vertex (iv) incident vertex

10. Every point of a Digraph lies in exactly ____ components. _____
(i) One component (ii) one strong (iii) one weak (iv) two weak

SECTION - B (25 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** Marks (5 x 5 = 25)

11(a) Prove that the number of vertices of odd degree in a graph in a graph is always even.

OR

(b) Prove that a simple graph with n vertices and k components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges.

12(a) Prove that every tree has either one or two centers.

OR

(b) Prove that there is one and only path between every pair of vertices in a tree T.

Cont...

13(a) Prove that Kuratowski's second graph is non-planar.

OR

(b) Explain elementary reduction.

14(a). Explain adjacency matrix with an example.

OR

(b) Explain circuit matrix.

15(a) Prove that the determinant of every square sub matrix of A, the incidence matrix of a digraph is 1,-1 or 0.

OR

(b) If A(G) is an incidence matrix of a connected graph G with n vertices then prove that the rank of A(G) is n-1.

SECTION -C (40 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 8 = 40)

16 (a) Prove that a connected graph G is an Euler graph iff all vertices of G are of even degree.

OR

(b) Explain Konigsberg bridge problem.

17(a). Prove that a tree with n vertices has (n-1) edges.

OR

(b) Explain rooted and binary trees.

18 (a) State and prove Euler's formula.

OR

(b) Prove that the complete graph of five vertices is non-planar.

19 (a) If B is a circuit matrix of a connected graph G with e edges and n vertices then prove that rank of B= e-n+1. And show that If G is a disconnected graph with k-components, e edges and n vertices then rank of B= $\mu = e - n + k$

OR

(b) Prove that (i) Every circuit has an even number of edges in common with any cut set.
(ii) Define fundamental circuits.

20 (a) (i)Define equivalence graph and (ii) Incidence matrix.

OR

(b) Explain types of Digraph.

Z-Z-Z END