**20MAU20/18MAU19**

**PSG COLLEGE OF ARTS & SCIENCE**
(AUTONOMOUS)

**BSc DEGREE EXAMINATION MAY 2022**
(Sixth Semester)

Branch – **MATHEMATICS**

**COMPLEX ANALYSIS**

Time: Three Hours Maximum: 75 Marks

**SECTION-A (10 Marks)**
Answer **ALL** questions
**ALL** questions carry **EQUAL** marks (10 x 1 = 10)

1 The function $f(z)=\bar{z}$ is
(i) differentiable at 0 only (ii) differentiable at any point z except 0
(iii) differentiable at any point z (iv) no where differentiable

2 A harmonic conjugate of $u=e^x\cos y$ is
(i) $e^x\cos y$ (ii) $e^{-x}\cos y$
(iii) $e^x\sin y$ (iv) $e^{-x}\sin y$

3 The fixed points of the transformation $w=az$ where $|a|=1$ are
(i) $\infty$ (ii) 0 and $\infty$
(iii) 1 and -1 (iv) i and -i

4 The bilinear transformation which maps $\infty, i, 0$ to $0, i, \infty$ is of the form
(i) $\frac{1}{z}$ (ii) $\frac{-1}{z}$
(iii) $\frac{i}{z}$ (iv) $\frac{-i}{z}$

5 The value of the integral $\int_C x\,dz$ where $C$ is the circle $|z|=r$ is
(i) $\pi r^2$ (ii) $2\pi r^2$
(iii) $i\pi r$ (iv) $i\pi r^2$

6 The value of the integral $\int_C \frac{1}{z-3}dz$ where $C$ is the circle $|z-2|=5$ is
(i) $\pi i$ (ii) $2\pi i$
(iii) $3\pi i$ (iv) $4\pi i$

7 The zeros of $f(z)=\frac{z^3-1}{z^2+1}$ are
(i) $i$ and $-i$ (ii) 1 and -1
(iii) 1, $\frac{-1+i\sqrt{3}}{2}$ and $\frac{-1-i\sqrt{3}}{2}$ (iv) 1, $\frac{-1+\sqrt{3}}{2}$ and $\frac{-1-\sqrt{3}}{2}$

**Cont...**

8 The singularity of the function $f(z)=\frac{z^2}{1+z}$ is classified as
(i) $z=-1$, a removable singularity
(ii) $z=-1$, a pole of order 1
(iii) $z=-1$, an essential singularity
(iv) $z=1$, a pole of order 1

9 The residue of $f(z)=\cot z$ at $z=0$ is
(i) 1 (ii) 0
(iii) $2\pi i$ (iv) $i$

10 The value of the integral $\int_C \frac{1}{2z+3}dz$ where $C$ is $|z|=1$ is
(i) $\pi i$ (ii) $2\pi i$
(iii) $4\pi i$ (iv) 0

## SECTION - B (25 Marks)

Answer **ALL** questions
**ALL** questions carry **EQUAL** Marks (5 x 5 = 25)

11 a Derive the Cauchy-Riemann equations in polar coordinates.
OR
b Define a conformal mapping. Also determine the angle of rotation and scale factor at the point $z=1+i$ under the mapping $w=z^2$.

12 a Find the image of the strip $2<x<3$ under the transformation $w=\frac{1}{z}$.
OR
b Prove that a bilinear transformation preserves inverse points.

13 a State and prove Cauchy's theorem for multiply connected regions.
OR
b State and prove Liouville's theorem.

14 a Find the Laurent's series for $\frac{z}{(z+1)(z+2)}$ about $z=-2$.
OR
b Define an isolated singularity and their types.

15 a State and prove Cauchy's residue theorem.
OR
b Use Contour integration evaluate $\int_0^{2\pi}\frac{1}{13+5\sin\theta}d\theta$.

## SECTION -C (40 Marks)

Answer **ALL** questions
**ALL** questions carry **EQUAL** Marks (5 x 8 = 40)

16 a State and prove the sufficient condition for differentiability of complex functions.

OR

b Let $f$ be an analytic function defined in a region $D$. Let $z_0 \in D$. If $f'(z_0) \neq 0$, then prove that $f$ is conformal at $z_0$.

17 a Determine the bilinear transformation which maps 0, 1, $\infty$ into i, -1, -i respectively. Under this transformation show that the interior of the unit circle of the z-plane maps onto the half plane left to the v axis (left half of the w-plane).

OR

b Prove that any bilinear transformation which maps the real axis onto unit circle $|w|=1$ can be written in the form $w = e^{i\lambda}\left(\frac{z-\alpha}{z-\bar{\alpha}}\right)$ where $\lambda$ is real. Further prove that this transformation maps the upper half plane $\operatorname{Im} z \geq 0$ onto the unit circular disc $|w| \leq 1$ iff $\operatorname{Im} \alpha > 0$.

18 a State and prove Cauchy's theorem.

OR

b Let $f$ be analytic inside and on a simple closed curve $C$. Let $z$ be any point inside $C$. Then prove that $f'(z) = \frac{1}{2\pi i}\int_C \frac{f(\zeta)}{(\zeta - z)^2} d\zeta$.

19 a State and prove Taylor's theorem.

OR

b Expand $f(z) = \frac{z}{(z-1)(2-z)}$ in a Laurent's series valid for (i) $|z|<1$ (ii) $1<|z|<2$ (iii) $|z|>2$ and (iv) $0<|z-2|<1$.

20 a Evaluate $\int_{-\infty}^{\infty} \frac{x^2 - x + 2}{x^4 + 10x^2 + 9} dx$.

OR

b Evaluate $\int_0^{\infty} \frac{x \sin x}{x^2 + a^2} dx$

Z-Z-Z END