20MAU01/18MAU01

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2022
(First Semester)

Branch – **MATHEMATICS**

CALCULUS - I

Time: Three Hours Maximum: 50 Marks

SECTION-A (5 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 1 = 5)

1. The curvature of a curve is __________ where T is the unit tangent vector.
   a) $k = \left|\frac{ds}{dT}\right|$ b) $k = \left|\frac{dT}{ds}\right|$
   c) $k = \frac{dT}{ds}$ d) $k = \frac{ds}{dT}$
2. The _________ curves of a function f of two variables are the curves with equations $f(x, y) = k$, where k is a constant.
   a) family b) level
   c) angle d) concentric
3. A point (a,b) is called a _______ point of f if $f_x(a, b) = 0$ and $f_y(a, b) = 0$.
   a) stationary b) constant
   c) vertex d) critical
4. The double riemann sum is ___________.
   a) $\sum_{i=1}^{m}\sum_{j=1}^{n} f(x_{ij}^*, y_{ij}^*)\,\Delta A$ b) $\sum_{i=1}^{m}\sum_{j=1}^{n} f(x_{ij}, y_{ij})\,\Delta A$
   c) $\sum_{i=1}^{m}\sum_{j=1}^{n} f(x_{ij}^*, y_{ij}^*)\,\nabla A$ d) $\sum_{i=1}^{m}\sum_{j=1}^{n} f(x_{ij}, y_{ij})\,\nabla A$.
5. In three dimensions, the cylindrical coordinates is similar to __________ coordinates and gives convenient descriptions of some surfaces and solids.
   a) spherical b) cartesian
   c) polar d) n-dimensional

SECTION - B (15 Marks)

Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 3 = 15)

6 a Find $\lim_{t\to 0} r(t)$, where $r(t) = (1 + t^3)i + te^{-t}j + \frac{\sin t}{t}k$.

OR

b Find the parametric equations for the tangent line to the helix with parametric equations $x = 2\cos t, y = \sin t, z = t$ at the point $(0,1,\frac{\pi}{2})$.

7 a Find the domain and range of $g(x, y) = \sqrt{9 - x^2 - y^2}$

OR

b Calculate f_{xxyz} if $f(x, y, z) = \sin(3x + y + z)$.

8 a If $z = x^2y + 3xy^4$, where $x = sin2t$ and $y = cost$, find $\frac{dz}{dt}$ when t=0.

OR

b Find the directional derivative of the function $f(x, y) = x^2y^3 - 4y$ at the point (2,-1) in the direction of the vector $v = 2i + 5j$.

Cont...

9 a If $R = \{(x,y) | -1 \le x \le 1, -2 \le y \le 2\}$, evaluate the integral $\int\int_R \sqrt{1-x^2}\, dA$.

OR

b Evaluate the iterated integral $\int_0^1 \int_x^1 sin(y^2)\, dydx$.

10 a Describe the surface whose equation in cylindrical coordinates is z=r.

OR

b Use a triple integral to find the volume of the tetrahedron T bounded by the planes $x + 2y + z = 2;\ x = 2y, x = 0, and\ z = 0$.

SECTION -C (30 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 6 = 30)

11 a Find a vector function that represents the curve of intersection of the cylinder $x^2 + y^2 = 1$ and the plane $y + z = 2$.

OR

b Find the curvature of the parabola $y = x^2$ at the points (0,0), (1,1) and (2,4).

12 a Find $\lim_{(x,y)\to(0,0)} \frac{3x^2y}{x^2+yz}$ if it exists.

OR

b Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if z is defined implicitly as a function of x and y by the equation $x^3 + y^3 + z^3 + 6xyz = 1$.

13 a If f is a differentiable function of x and y, then prove that f has a directional derivative in the direction of any unit vector $u = \langle a, b \rangle$ and $D_u f(x,y) = f_x(x,y)a + f_y(x,y)b$.

OR

b Find the local maximum and minimum values and the saddle points of $f(x,y) = x^4 + y^4 - 4xy + 1$.

14 a Evaluate $\int\int_D (x + 2y)\, dA$, where D is the region bounded by the poarabolas $y = 2x^2$ and $y = 1 + x^2$.

OR

b Find the volume of the tetrahedron bounded by the planes $x + 2y + z = 2, x = 2y, x = 0, and\ z = 0$.

15 a Find the center of mass of a solid of constant density that is bounded by the parabolic cylinder $x = y^2$ and the planes x=z, z=0, and x=1.

OR

b Find the area of the part of the paraboloid $z = x^2 + y^2$ that lies under the plane z=9.

Z-Z-Z END