PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2022
(Fourth Semester)

Branch – MATHEMATICS

NUMBER THEORY

Time: Three Hours   Maximum: 75 Marks

SECTION-A (10 Marks)
Answer ALL questions
ALL questions carry EQUAL marks   (10 x 1 = 10)

1 If $a \in \mathbb{N}$ then its successor a^+ = ___.
  (i) a   (ii) a–1   (iii) a+1   (iv) 0.

2 Two integers a and b are said to be ____ of each other if a|b and b|a.
  (i) divides   (ii) associates   (iii) commutes   (iv) H.C.F.

3 Which are the following are called Siamese twins?
  (i) 2, 3   (ii) 3, 5   (iii) 7, 11   (iv) 13, 17.

4 If p is a prime then $d(p)$ =?, $\sigma(p)$ =?
  (i) 1, p   (ii) 2, p   (iii) 2, p+1   (iv) 2, p+2.

5 "Every even number ≥ 4 is a sum of two primes" this conjecture is due to ____.
  (i) Gold bach   (ii) Dirichlet   (iii) Euler   (iv) Fermat.

6. The magic number of 4623 is ___.
  (i) 2   (ii) 3   (iii) 4   (iv) 6.

7 "If n is a prime number and m be any integer then $m^n \equiv m \pmod{n}$" this theorem is known as ____ theorem.
  (i) Fermat   (ii) Little Fermat
  (iii) Wilson   (iv) Wolstenholme.

8 If $(a, n) = 1$ then $a^{\Phi(n)} \equiv ? \pmod{n}$.
  (i) 0   (ii) –1   (iii) 1   (iv) a.

9 Let (x, y, z) be a Pythagorean triple. If k divides any two of x, y, and z then ____.
  (i) k divides the third   (ii) k does not divides the third
  (iii) k may divides the third   (iv) none of these.

10 For every integer n > 2 the equation $x^n + y^n = z^n$ has no solutions in integers is called ____ theorem.
  (i) Fermat's little   (ii) Fermat
  (iii) Warning   (iv) Fermat's big.

SECTION - B (25 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** Marks   (5 x 5 = 25)

11.a) Prove that ">" is an order relation in $\mathbb{N}$.

OR

b) Find the H.C.F of 963 and 657 and express it in the form $963x + 657y$ in two different ways.

12.a) State and prove Euclid's theorem.

OR

b) Verify that 220 and 284 are amicable numbers.

13.a) Prove: If $a \geq 2$ and $a^n + 1$ is prime then a is even and n is a power of 2.

OR

b) Solve $3x \equiv 1 \pmod{125}$.

14.a) Prove that the ninth power of every number is of the form $19m$ or $19m \pm 1$.

OR

b) Find the last two digits of 7^{355}.

15.a) State and prove Fermat's last theorem.

OR

b) Prove that any product of representable numbers is representable.

SECTION -C (40 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** Marks   (5 x 8 = 40)

16.a) Prove $3^{2n-1} + 2^{n+1} = M(7)$ *by using* Mathematical Induction .

OR

b) Prove that any two integers a and b have a unique greatest common divisor.

17.a) State and prove unique factorization theorem.

OR

b) Prove that any two consecutive Fibonacci numbers are coprimes.

18.a) i. Prove: If $n > 1$ and $a^n - 1$ is prime then a = 2 and n is prime.   (5)
ii. Prove that the square of an odd number is of the form $8n+1$.   (3)

OR

b) Solve: $x \equiv 2 \pmod 3$, $x \equiv 4 \pmod 5$, $x \equiv 6 \pmod 7$.

19.a) If a and b are relatively prime to 91 then prove that $a^{12} - b^{12}$ is divisible by 91.

OR

b) State and prove Wolstenholme theorem.

20.a) Find all solutions of $x^2 + y^2 = z^2$ with $40 \leq z \leq 50$.

OR

b) Find all of the essentially different representations of 5525 as a sum of two squares.

Z-Z-Z   END