20MAU13/18MAU13

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2022
(Fifth Semester)

Branch – **MATHEMATICS**

ALGEBRA

Time: Three Hours — Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 x 1 = 10)

1. If $n(S) = n$, then $n(A(S)) =$
   (i)n (ii)2^n (iii)n^2 (iv)$n!$
2. The set of all integers is not a ------- under multiplication
   (i) abelian group (ii) group (iii) semigroup (iv)cyclic group
3. If H, K are subgroups of the abelian group G, then HK is a------------ of G
   (i)Cyclic group (ii) prime (iii) subgroup (iv) abelian group
4. Every subgroup of an abelian group is ----
   (i) cyclic (ii) semigroup (iii) quotient group (iv) normal
5. If $\theta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 3 & 5 & 6 & 4 \end{pmatrix}$ the orbit of 3 is
   (i)(1,2) (ii)(3) (iii)(1,2,3,4) (iv)(5,3)
6. The number of automorphisms of a cyclic group of order n is ---
   (i)$\emptyset(n+1)$ (ii)$\emptyset(n)$ (iii) $\emptyset(n-1)$ (iv)$\emptyset(n+2)$
7. If R is the ring of integers $mod6$ then R is a
   (i) field (ii)non commutative ring
   (iii) integral domain (iv)commutative ring
8. A finite integral domain is a -----
   (i) Euclidean (ii) field (iii) prime (iv) normal
9. R be a commutative ring with unit element the only ideals are --------
   (i)$\{0\}$ and R (ii)$\{1\}$ and R (iii)$\{0\}$and $\{1\}$ (iv)$\{0\}$ and $\{2\}$
10. If an ideal U of a ring R contains a unit element of R then ------
   (i) $U \neq R$ (ii)U=1 (iii)U=R (iv)R=1

SECTION - B (25 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** Marks (5 x 5 = 25)

11. a). Prove that, if S has more that two elements we can find two elements σ, τ in $A(S)$ such that $\sigma \cdot \tau \neq \tau \cdot \sigma$

(OR)

b). If G is a group in which $(a \cdot b)^i = a^i \cdot b^i$ for three consecutive integers i for all $a, b \in G$, show that G is abelian.

12. a) Prove that if N is a normal subgroup of G if and only if $gNg^{-1} = N$ for every $g \in G$.

(OR)

b).Prove that if $\emptyset$ is a homomorphism of G into $\bar{G}$ with kernel K, then K is normal subgroup of G.

Cont...

13. a) Let G be a group, for $g \in G$ $T_g: G \to G$ by $xT_g = g^{-1}xg$ for all $x \in G$ Prove that T_g is an automorphism.

(OR)

b). $I(G) \approx G/Z$ when $I(G)$ is the group of inner automorphisms of G and show that Z is the center of G .

14. a.) If R is a ring, then for all $a, b \in R$ then prove that

(i) $a0 = 0a = 0$

(ii) $a(-b) = (-a)b = -(ab)$

(iii) $(-a)(-b) = ab$

(OR)

b) If F is a field, prove its only ideals are $\{0\}$ and F itself

15. a). Prove that a Euclidean ring possesses a unit element (or)

b). Let R be a Euclidean ring. Suppose that for $a, b, c \in R, a|bc$ but $(a, b) = 1$.Then show that $a|c$.

SECTION -C (40 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 8 = 40)

16. a). Let G be the set of all 2×2 matrices $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ where a, b, c, d are real numbers such that $ad - bc \neq 0$.Then show that G is a non-abelian group under product of matrices .

(OR)

b. State and construct Lagrange's theorem.

17. a). If H and K are finite subgroups of G of orders $o(H)$ and $o(K)$ respectively, then show that $o(HK) = \frac{o(H)o(K)}{o(H \cap K)}$.

(OR)

b) State and Prove sylow's theorem.

18. a) State and prove Cayley theorem.

(OR)

b) (i)Find the orbit of the following permutation.

(1) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 5 & 6 & 2 & 4 & 8 & 3 & 1 & 7 \end{pmatrix}$

(2) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 1 & 3 & 6 & 2 & 4 \end{pmatrix}$

(ii) Express the permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 6 & 4 & 1 & 8 & 2 & 5 & 7 \end{pmatrix}$ as a product of disjoint cycles and then as a product of transposition.

19. a.) Analyze a finite integral domain is a field.

(Or)

b) (i) Interpret the homomorphism $\emptyset$ of R into R' is an isomorphism if and only if $I(\emptyset) = \{0\}$

(ii)If U is an ideal of R and $1 \in U$ show that $U = R$.

20. a) If R is a commutative ring with unit element and M is an ideal of R, then M is a maximal ideal of R if and only if R/M is a field

(OR)

b). Let R be a Euclidean ring and $a, b \in R$. If $b \neq 0$ is not a unit in R then determine $d(a) < d(ab)$

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