TOTAL PAGES: 2
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PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2022
(Second Semester)

Branch – **COMPUTER SCIENCE WITH DATA ANALYTICS**

DISCRETE STRUCTURES AND GRAPH THEORY

Time: Three Hours — Maximum: 50 Marks

SECTION-A (5 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks
One question from each unit (with four choices) — (5 x 1 = 5)

1. A product of the variables and their negations in a formula is ________
(i) elementary product (ii) elementary sum
(iii) normal form (iv) minterms.

2. A relation R in X is said to be a ________ if it is reflexive and symmetry.
(i) equivalence relation (ii) compatibility relation.
(iii) composite relation (iv) Binary relation.

3. A mapping of $f: X \rightarrow Y$ is called __________ (surjective, a surjection) if the range $R_f = y$.
(i) one – to – one (ii) one – to – one on to
(iii) onto (iv) None of them.

4. A closed walk in which no vertex appears more than once is called a ________
(i) path (ii) train (iii) terminal (iv) circuit

5. A _______ is a connected graph without any circuits.
(i) rooted tree (ii) tree (iii) binary tree (iv)vertex.

SECTION - B (15 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks
One question from each unit with either or type — (5 x 3 = 15)

6. a) Show that the formula $Q \vee (P \wedge \neg Q) \vee (\neg P \wedge \neg Q)$ is a tautology.

(OR)

b) Show I_{12} : $\neg Q, P \rightarrow Q \Rightarrow \neg P$.

7. a) Let x ={1, 2, 3, 4} and R = { < x, y> | x > y}. Draw the grath of R and also give its matrix.

(OR)

b) Let R and S be two relations on a set of positive integers I:
$R = \{ \langle x, 2x \rangle \mid x \in I\}$ $S = \{ \langle x, 7x \rangle \mid x \in I\}$
Find RoS, RoR, RoRoR, and Ro So R.

8. a) Show that the functions $f(x) = x^3$ and $g(x) = x^{1/3}$ for $x \in R$ are inverses of one another .

(OR)

b) Show that $2^n < n!$ for $n \geq 4$.

9. a) Show that the number of vertices of odd degree in a graph is always even.

(OR)

b) Define finite graph, isolated vertex, pendant vertex.

Cont...

10. a) Show that a graph G with n vertices, n -1 edges, and no circuits is connected.

(OR)

b) If in a graph G there is one and only one path between every pair of vertices, then show that G is a tree.

SECTION -C (30 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** Marks
One question from each unit with either or type (5 x 6 = 30)

11. a) Obtain the Principal disjunctive normal forms of
(i) $\neg P \vee Q$
(ii) $(P \wedge Q) \vee (\neg P \wedge R) \vee (Q \wedge R)$.

(OR)

b) Obtain disjunctive normal form of
(i) $P \wedge (P \rightarrow Q)$
(ii) $\neg(P \vee Q) \square (P \wedge Q)$.

12. a) Let $R = \{\langle 1, 2\rangle, \langle 3, 4\rangle, \langle 2, 2\rangle\}$ and
$S = \{\langle 4, 2\rangle, \langle 2, 5\rangle, \langle 3, 1\rangle, \langle 1, 3\rangle\}$.
Find RoS, SoR, Ro (SoR), (RoS) oR, RoR, SoS and RoRoR.

(OR)

b) Illustrate properties of Binary relations in a set.

13. a) Let $X = \{1, 2, 3\}$ and f, g, h and s be functions from X to X given by
$f = \{\langle 1, 2\rangle, \langle 2, 3\rangle, \langle 3, 1\rangle\}$, $g = \{\langle 1, 2\rangle, \langle 2, 1\rangle, \langle 3, 3\rangle\}$
$h = \{\langle 1, 1\rangle, \langle 2, 2\rangle, \langle 3, 1\rangle\}$, $s = \{\langle 1, 1\rangle, \langle 2, 2\rangle, \langle 3, 3\rangle\}$
Find fog; gof; fohog; sog; gos, sos and fos.

(OR)

b) Prove that $n^3 + 2n$ is divisible by 3.

14. a) Prove that a simple graph with n vertices and k components can have at most $(n - k)(n - k + 1) / 2$ edges.

(OR)

b) Describe Konigsberg Bridge problem.

15. a) Prove that every tree has either one or two centers.

(OR)

b) Prove that a tree with n vertices has n -1 edges.

Z-Z-Z END