18MCU23

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2022
(Fifth Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

LINEAR ALGEBRA

Time: Three Hours Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 x 1 = 10)

1. If v is a vector space, then the elements of v and the elements of F are called ______.
   (i) Basis and dimensions (ii) Linear Combination and space
   (iii) Vectors and Scalars (iv) Linearly dependent and linearly independent

2. If $S = \{v_1, v_2, ..., v_n\}$, a set of vectors is a finite dimensional vector space v, then S is called a basis for v if ....
   $(i)\ S\ spans\ v$ (ii) S is linearly independent
   (iii) either (i) or (ii) (iv) both (i) and (ii)

3. What is the dimension of zero vector space?
   (i) Not defined(ii) 1 (iii) 0 (iv) infinite

4. Let $S = \{(-1,0,1), (2,1,4)\}$. What is the value of k for which the vector $(3k + 2,3,10)$ belongs to the linear span of S?
   (i)8 (ii) 4 (iii) -2 (iv) 2

5. Identify $(\alpha u + \beta\ v)(T_1T_2)$ is
   (i) $\alpha(u(T_1T_2)) + \beta(v(T_1T_2))(ii)\alpha(u(T_1T_2)) + \beta(u(T_1T_2))$
   (iii) $\alpha(u((T_2)) + \beta(v(T_1))$ (iv) $\alpha\ u(T_1T_2) + \beta v(T_1T_2)$

6. If $T \in A(v)$ then the range of T, vT is defined by ......
   $(i)\{vT/v \in V\}$ (ii) $\{T/v \in VT\}$ (iii) $\{vT/v \in T\}$ (iv) $\{tv/t \in T\}$

7. What is the maximum positive rank of an $m \times n$ matrix A that is not square?
   (i) rank $(A) \geq \min(m,n)$ (ii) rank $(A) \leq \min(m,n)$
   (iii) rank $(A) = \max(m,n)$ (iv) rank $(A) = \max(m,n)$

8. The transition matrices are .....
   (i) not at all invertible (ii) invertible always
   (iii) invertible sometimes (iv) data not complete

9. Find the wrong one from the following statements:
   (i) If A is diagonalizable and invertible, then A^{-1} is diagonalizable.
   (ii) If A is diagonalizable, then A^T is diagonalizable.
   (iii) If every eigenvalue of a matrix A has algebraic multiplicity 1, then A is diagonalizable.
   (iv) An $n \times n$ matrix with fewer than n distinct eigenvalues is not diagonalizable.

10. If 0 is an eigenvalue of a matrix A, then the set of columns of A is ________.
    (i) Linearly independent or linearly dependent
    (ii) Linearly dependent always
    (iii) Linearly independent always
    (iv) Cannot be determined

SECTION - B (25 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** Marks (5 x 5 = 25)

11. a) If V is the internal direct sum of $U_1, ..., U_n$, then show that V is isomorphic to the external direct sum of $U_1, ..., U_n$.
    (OR)
    b) Show that $L(s)$ is a subspace of V.

Cont...

12. a) Show that $A(A(W)) = W$.

(OR)

b) Describe about inner product space with an example.

13. a) If V is finite dimensional over F, then analyze $T \in A(V)$ is invertible if and only if the constant term of the minimal polynomial for T is not 0.

(OR)

b) If $\lambda \in F$ is a characteristic root of $T \in A(V)$, then show that for any polynomial $q(x) \in F[x], q(x)$ is a characteristic root of $q(T)$.

14. a) Let $A = \begin{bmatrix} 1 & -3 & -4 \\ -4 & 6 & -2 \\ -3 & 7 & 6 \end{bmatrix}$ and $b = \begin{bmatrix} 3 \\ 3 \\ -4 \end{bmatrix}$. Calculate whether b is the column space of A.

(OR)

b) If $A = \begin{bmatrix} 6 & 1 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}$. Verify $\det AB = (\det A)(\det B)$.

15. a) Let $A = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}, u = \begin{bmatrix} 6 \\ -5 \end{bmatrix}, v = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$. State whether u *and* v are eigenvalue of A.

(OR)

b) Bring out the eigenvalue of $A = \begin{bmatrix} 2 & 3 \\ 3 & -6 \end{bmatrix}$.

SECTION -C (40 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 8 = 40)

16. a) If $V_1, V_2, \dots V_n$ is a basis of V over F and if $w_1, w_2, \dots, w_n$ in V are linearly independent over F, then justify $m \le n$.

(OR)

b) If V is finite dimensional and if W is a subspace of V, classify $W$is finite dimensional $\dim W \le \dim V$ and $\dim \frac{V}{W} = \dim V - \dim W$.

17. a) If $u, v \in V$, then show that $|(u, v)| \le \|u\|\|v\|$.

(OR)

b) If V is a finite dimensional inner product space and if W is a subspace of V, then justify $V = W + W^{\perp}$. More particularly, prove that V is the direct sum of W *and* $W^{\perp}$.

18. a) If A is an algebra, with unit element over F, then analyze A is isomorphic to a sub algebra of $A(V)$ for some vector space V over F.

(OR)

b) If $\lambda \in F$ is a characteristic root of $T \in A(V)$, show that λ is a root of the minimal polynomial of T. In particular, prove that T only has a finite number of characteristic roots in F.

19. a) Calculate the inverse of the matrix $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 4 & -3 & 8 \end{bmatrix}$, if it exists.

(OR)

b) Discover $\det A$, *where* $A = \begin{bmatrix} 0 & 1 & 2 & -1 \\ 2 & 5 & -7 & 3 \\ 0 & 3 & 6 & 2 \\ -2 & -5 & 4 & -2 \end{bmatrix}$.

20. a) Discover the characteristic equation of $A = \begin{bmatrix} 5 & -2 & 6 & -1 \\ 0 & 3 & -8 & 0 \\ 0 & 0 & 5 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

(OR)

b) Let $A = \begin{bmatrix} 7 & 2 \\ -4 & 1 \end{bmatrix}$. Discover a formula for A^k, given that $A = PDP^{-1}$, where $P = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}$ and $D = \begin{bmatrix} 5 & 0 \\ 0 & 3 \end{bmatrix}$.

Z-Z-Z END