22MCU101

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2022
(First Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

CALCULUS

Time: Three Hours | Maximum: 50 Marks

SECTION-A (5 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 1 = 5)

1 The distance from the point $P(2,-1,7)$ to the point $Q(1,-3,5)$ is
   (i) $\sqrt{15}$ (ii) $\sqrt{10}$
   (iii) 3 (iv) 9

2 $\lim_{(x,y)\to(1,2)}(x^2y^3 - x^3y^2 + 3x + 2y) =$ ______.
   (i) 10 (ii) 11
   (iii) 9 (iv) 12

3 If $f(x,y,z) = x\sin yz$ then $grad(f) =$ ______.
   (i) $(\sin yz, xz\sin xz, xy\sin yz)$ (ii) $(\cos yz, xz\cos yz, xy\cos yz)$
   (iii) $(\sin yz, xz\cos yz, xy\cos yz)$ (iv) $(\sin yz, xz, xz\sin yz, xy\cos yz)$

4 $\int_0^2\int_1^2 (x-3y^2)\,dy\,dx =$ ____________.
   (i) -12 (ii) 12
   (iii) 21 (iv) -21

5 If $x = u^2 - v^2,\ y = 2uv$ then $\frac{\partial(x,y)}{\partial(u,v)} =$ ______.
   (i) $u^2 + v^2$ (ii) $4uv$
   (iii) $4u^2 - 4v^2$ (iv) $4u^2 + 4v^2$

SECTION - B (15 Marks)

Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 3 = 15)

6a) Find the angle between the vectors $a = (2,2,-1)$ *and* $b = (5,-3,2)$.

OR

b) Find the length of the arc of the circular helix with vector equation $r(t) = \cos t\ \vec{i}\_\sin t\ \vec{j} + t\ \vec{k}$ from the point $(1,0,0)$ to the point $(1,0,2\pi)$.

7a) Find the level curves of the function $f(x,y) = 6 - 3x - 2y$ for the values $k = -6,0,6,12$.

OR

b) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if z is defined implicitly as a function of x and y by the equation $x^3 + y^3 + z^3 + 6xyz = 1$.

8a) If $z = e^x \sin y$ where $x = st^2$ and $y = s^2t$ find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$.

OR

b) Find the direction derivative $D_u f(x,y)$ if $f(x,y) = x^3 - 3xy + 4y^2$ and u is the unit vector given by angle $\theta = \frac{\pi}{6}$ what is $D_u f(1,2)$.

9a) Use the midpoint rule with $m = n = 2$ to estimate the value of the integral $\iint_R (x-3y^2)\,dA$ where $R = \{(x,y)/0 \le x \le 2,,1 \le y \le 2\}$.

OR

Cont...

b) Evaluate $\iint_D (x+2y)\, dA$ where D is the region bounded by the parabolas $y = 2x^2$ and $y = 1 + x^2$.

10a) Evaluate the triple integral $\iiint_B xyz^2\, dV$, where B is the rectangular box $B = \{(x,y,z) / 0 \le x \le 1,\ -1 \le y \le 2,\ 0 \le z \le 3$.

OR

b) The point $(2, \frac{\pi}{4}, \frac{\pi}{3})$ is given in spherical coordinates , plot the point and find its rectangular coordinates.

SECTION -C (30 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 6 = 30)

11a Find a vector perpendicular to the plane that passes through the points $P(1,4,6)$, $Q(-2,5,-1)$, $R(1,-1,1)$.

OR

b) Find the curvature of the twisted cubic $r(t) = < t, t^2, t^3 >$ at a general point and at $(0,0,0)$.

12a) (i) Find $\lim_{(x,y)\to(0,0)} \frac{3x^2 y}{x^2 + y^2}$ if it exists .

(ii) If $f(x,y) = \frac{xy^2}{x^2 + y^2}$, then does $\lim_{(x,y)\to(0,0)} f(x,y)$ exist?

OR

b) (i) Find the second Partial derivatives of $f(x,y) = x^3 + x^2y^3 - 2y^2$.

(ii) Calculate f_{yz} if $f(x,y,z) = \sin(3x + yz)$.

13a)(i) If $g(s,t) = f(s^2 - t^2, t^2 - s^2)$ and f is differentiable show that g satisfies the equation $t\frac{\partial g}{\partial s} + S\frac{\partial g}{\partial t} = 0$.

(ii) If $z = f(x,y)$ has continuous second order partial derivatives and $x = t^2 + s^2$ and $y = 2rs$ then find $\frac{\partial z}{\partial r}$ and $\frac{\partial^2 z}{\partial r^2}$.

OR

b) Find the local maximum and minimum values and saddle points of $f(x,y) = x^4 + y^4 + 4xy + 1$.

14a) Evaluate $\iint_R (3x + 4y^2)\, dA$ where D is the region in the upper half-plane bounded by the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

OR

b) Evaluate the mass and centre of mass of a triangular lamina with vertices $(0,0), (1,0)$ and $(0,2)$ if the density function is $\rho(x,y) = 1 + 3x + y$.

15a) Find center and mass of the solid of constant density that is bounded by the parabolic cylinder $x = y^2$ and the planes $x = z, z = 0$ and $x = 1$.

OR

b) (i) Plot the point with cylindrical coordinates $(2, \frac{2\pi}{3}, 1)$ and find its rectangular coordinates.

(ii) Find cylindrical coordinates of the point with rectangular coordinates . $(3,-3,-7)$.

Z-Z-Z END

22MCU102

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2022
(First Semester)

Branch – **MATHEMATICS WITH COMPUITER APPLICATIONS**

ORDINARY DIFFERENTIAL EQUATIONS AND LAPLACE TRANSFORMS

Time: Three Hours Maximum: 50 Marks

SECTION-A (5 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 1 = 5)

1 A homogeneous first order differential equation is
(i) $\frac{dy}{dx}=F\left(\frac{y}{x}\right)$ (ii) $\frac{dx}{dy}=F\left(\frac{y}{x}\right)$
(iii) $\frac{dy}{dx}=F\left(\frac{x}{y}\right)$ (iv) $\frac{dx}{dy}=F\left(\frac{x}{y}\right)$

2 What is a repeated root in 2nd order differential equation
(i) $x(y)=(c_1+c_2x)e^{r_1x}$ (ii) $y(x)=(c_1+c_2y)e^{r_1y}$
(iii) $y(x)=(c_1+c_2x)e^{r_1x}$ (iv) $x(y)=(c_1+c_2y)e^{r_1x}$

3 A capacitor with a capacitance of C is
(i) Ohms (ii) farads
(iii) Henries (iv) Voltage

4 $L(e^{at}f(t))$ is equal to
(i) $f(y-a)$ (ii) $f(s-a)$
(iii) $F(y-a)$ (iv) $F(s-a)$

5 What is a differentiation of transforms
(i) $L(tf(t))=F'(s)$ (ii) $L(-tf(t))=F'(t)$
(iii) $L(-tf(t))=F'(s)$ (iv) None

SECTION - B (15 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 3 = 15)

6 a Define velocity, acceleration and constant acceleration.

OR

b Solve the initial value problem $x^2\frac{dy}{dx}+xy=\sin x,\ y(1)=y_0$.

7 a Find the general solution of $2y''-7y'+3y=0$.

OR

b Show that the functions $y_1(x)=e^{-3x},\ y_2(x)=\cos 2x,\ y_3(x)=\sin 2x$ are linearly independent.

8 a Find a general solution of $(D^2+6D+13)^2y=0$.

OR

b Find a particular solution of $y''-4y=2e^{3x}$.

Cont...

9 a Show that $L\left(te^{at}\right)=\frac{1}{(s-a)^2}$.

OR

b Prove that $L\{af(t)+bg(t)\}=aL\{f(t)\}+bL\{g(t)\}$.

10 a Find $L\{t^2\sin kt\}$.

OR

b Find $L\{g(t)\}$ if $g(t)=\begin{cases}0 & if\ t\prec 3\\ t^2 & if\ t\geq 3\end{cases}$.

SECTION -C (30 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 6 = 30)

11 a Solve the initial value problem $\frac{dy}{dx}=2x+3,\ y(1)=2$.

OR

b Solve the differential equation $2xy\frac{dy}{dx}=4x^2+3y^2$.

12 a Let y_1 and y_2 be two solutions of the homogeneous linear equation in $y''+p(x)y'+q(x)y=0$ on the interval I. If c_1 and c_2 are constants, then the linear combination $y=c_1y_1+c_2y_2$ is a solution of $y''+p(x)y'+q(x)y=0$ 0n I.

OR

b It is evident that $y_p=3x$ is a particular solution of the equation $y''+4y=12x$ and that $y_c(x)=c_1\cos 2x+c_2\sin 2x$ is its complementary solution and satisfies the initial condition $y(0)=5, y'(0)=7$. Find the solution of the equation.

13 a Solve the IVP $y''-3y'+2y=3e^{-x}-10\cos 3x$; $y(0)=1,\ y'(0)=2$.

OR

b Find a particular solution of the equation $y''+y=\tan x$.

14 a Solve the IVP $x''-x'-6x=0$; $x(0)=2,\ x'(0)=-1$.

OR

b Solve the IVP $y''+4y'+4y=t^2$; $y(0)=y'(0)=0$.

15 a Prove that $L\{f(t)*g(t)\}=L\{f(t)\}.L\{g(t)\}$.

OR

b Prove that $F(s)=\frac{1}{1-e^{-ps}}\int_0^p e^{-st}f(t)dt$.

Z-Z-Z END

19MCU05

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2022
(Second Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

ANALYTICAL GEOMETRY OF 3D AND VECTOR CALCULUS

Time: Three Hours Maximum: 50 Marks

SECTION-A (5 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 1 = 5)

1. The length of the perpendicular from the point (4,3,0) to the plane $2x + 3y + \sqrt{3}z + 3 = 0$ is ________
(i) $\frac{1}{4}$ (ii) 5 (iii) $\frac{3}{4}$ (iv) $\frac{1}{5}$

2. The equation of the tangent plane at (0,0,1) to the sphere $x^2 + y^2 + z^2 = 1$ is ___
(i) $x = 0$ (ii) $y = 0$ (iii) $z = 1$ (iv) $x + y + z = 1$

3. A cone of second degree can be found to pass through ______concurrent lines.
(i) 5 (ii) 6 (iii) 4 (iv) 3

4. If C is the straight line joining (0,0,0) and (1,1,1) then $\int_C \boldsymbol{r}.\boldsymbol{dr}$ is _______
(i) $\frac{1}{2}$ (ii) 1 (iii) $\frac{3}{2}$ (iv) 2

5. Gauss' divergence theorem connects________.
(i) Line integral and double integral (ii) Line integral and surface integral
(iii) Double integral and surface integral (iv) Surface integral and volume integral

SECTION – B (15 Marks)
Answer **ALL** Questions
ALL questions carry **EQUAL** marks (5 x 3 = 15)

6. a) Find the distance between the parallel planes $x + 2y - 3z + 1 = 0$ and $2x + 4y - 4z + 5 = 0$.
OR
b) Find the perpendicular distance of $P(1,2,3)$ from the line $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$.

7. a) Find the radius and centre of the sphere $x^2 + y^2 + z^2 - 2x + 4y - 6z = 2$.
OR
b) Find the equation of the sphere through the circle $x^2 + y^2 + z^2 = 9, 2x + 3y + 4z = 5$ and the point (1,2,3).

8. a) Find the equation of the cone whose vertex is at the origin and which passes through the curve given by the equations $ax^2 + by^2 + cz^2 = 1, lx + my + nz = p$.
OR
b) Find the equation to the right circular cone whose vertex is $P(2,-3,5)$, axis PQ which makes equal angles with the axes and semi-vertical angle is 30°.

9. a) Evaluate $\int_C (2 + x^2y)\, ds$, where C is the upper half of the unit Circle $x^2 + y^2 = 1$.
OR
b) Evaluate $\int_C x^4 dx + xydy$, where C is the triangular curve consisting of the line segments from (0,0) to (1,0), from (1,0) to (0,1), and from (0,1) to (0,0).

19MCU05
Cont...

10. a) Find a parametric representation for the cylinder $x^2 + y^2 = 4 \ \ 0 \le z \le 1$.

OR

b) Compute the surface integral $\iint_S x^2 \, dS$, where S is the unit sphere $x^2 + y^2 + z^2 = 1$.

SECTION -C (30 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 6 = 30)

11. a) Find the volume of a tetrahedron in terms of the lengths of the three edges which meet in a point and of the angles which these edges make with each other in pairs.

OR

b) Show that the planes $ax + hy + gz = 0, hx + by + fz = 0, gx + fy + cz = 0$ have a common line of intersection if $\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$ and the direction ratios of the line satisfy the equations $\frac{l^2}{\partial\Delta/\partial a} = \frac{m^2}{\partial\Delta/\partial b} = \frac{n^2}{\partial\Delta/\partial c}$.

12. a) Find the equation of the circle circumscribing the triangle formed by the three points $(a, 0,0), (0, b, 0), (0,0, c)$. Also Obtain the co-ordinates of the centre of this circle.

OR

b) Show that the spheres $x^2 + y^2 + z^2 = 64$ and $x^2 + y^2 + z^2 - 12x + 4y - 6z + 48 = 0$ touch internally and find their point of contact.

13. a) Two cones pass through the curves $y = 0, z^2 = 4ax; x = 0, z^2 = 4$ and they have a common vertex. The plane $z = 0$ meets them in two conics that intersect in four concyclic points. Show that the vertex lies on the surface $z^2 \left(\frac{x}{a} + \frac{y}{b}\right) = 4(x^2 + y^2)$.

OR

b) Find the equation to the lines in which the planes $2x + y - z = 0$ cuts the cone $4x^2 - y^2 + 3z^2 = 0$.

14. a) (i) If $\boldsymbol{F}(x, y) = (3 + 2xy)\boldsymbol{i} + (x^2 - 3y^2)\boldsymbol{j}$, find a function f such that $\boldsymbol{F} = \nabla f$.
(ii) Evaluate the line integral $\int_C \boldsymbol{F}.\boldsymbol{dr}$, where C is the curve given by $\boldsymbol{r}(t) = e^t \sin t \, \boldsymbol{i} + e^t \cos t \, \boldsymbol{j} \ \ 0 \le t \le \pi$.

OR

b) (i) Show that $\boldsymbol{F}(x, y, z) = y^2 z^3 \boldsymbol{i} + 2xyz^3 \boldsymbol{j} + 3xy^2 z^2 \boldsymbol{k}$ is a conservative vector field.
(ii) Find a function f such that $\boldsymbol{F} = \nabla f$.

15. a) Find the tangent plane to the surface with parametric equations $x = u^2, y = v^2, z = u + 2v$ at the point $(1,1,3)$.

OR

b) Evaluate $\iint_S \boldsymbol{F}.\boldsymbol{dS}$, where $\boldsymbol{F}(x, y, z) = y\boldsymbol{i} + x\boldsymbol{j} + z\boldsymbol{k}$ and S is the boundary of the solid region E enclosed by the paraboloid $z = 1 - x^2 - y^2$ and the plane $z = 0$.

Z-Z-Z END

20MCU11

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2022
(Third Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**
DATA STRUCTURES USING C++

Time: Three Hours   Maximum: 50 Marks

SECTION-A (5 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks   (5 x 1 = 5)

1. What is C++?
   (i) C++ is an object oriented programming language
   (ii) C++ is a procedural programming language
   (iii) C++ supports both procedural and object oriented programming language
   (iv) C++ is a functional programming language

2. Which concept allows you to reuse the written code in C++?
   (i) Inheritance   (ii) Polymorphism
   (iii) Abstraction   (iv) Encapsulation

3. Which of the following correctly declares an array?
   (i) int array[10];   (ii) int array;
   (iii) array{10};   (iv) array array[10];

4. Which of the following is a linear data structure?
   (i) Array   (ii) AVL Trees
   (iii) Binary Trees   (iv) Graphs

5. When a pop() operation is called on an empty queue, what is the condition called?
   (i)Overflow   (ii)Underflow
   (iii)Syntax Error   (iv)Garbage Value

SECTION - B (15 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks   (5 x 3 = 15)

6 a. Explain function overloading?

OR

b. What is the role of protected access specifier?

7 a. Explain copy constructors.

OR

b. What is the purpose of using a destructor in C++?

8 a. Analyse Data structures operations.

OR

b. Describe Linear search traversing.

Cont...

9 a. Explain Polish Notation.

OR

b. Explain the operation of PUSH operation.

10 a. Explain the complexity of sorting algorithms.

OR

b. Explain Insertion sort with example.

SECTION -C (30 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 6 = 30)

11 a. Explain (i) Inline functions (ii) Member functions

OR

b. Write a C++ program add 2 members of two different classes using friend functions.

12 a. Explain (i) Multi level inheritance (ii) Multiple inheritance with examples.

OR

b. Discuss Pointers in C++.

13 a. Explain Binary search concept.

OR

b. Enumerate Multidimensional arrays.

14 a. Explain in detail about stack representation.

OR

b. Discuss Array representation of queues.

15 a. Survey about Merge sort.

OR

b. Describe Radix sort with example.

Z-Z-Z END

18MCU06

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2022
(Second Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

PROGRAMMING IN C

Time: Three Hours Maximum: 50 Marks

SECTION-A (5 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 1 = 5)

1 ____________ is a valid C expression.
(i) int my_var=3,000; (ii) int my_var=30;
(iii) int my var=3; (iv) int $my_var=300;

2 The value of EOF is ____________.
(i) 0 (ii) 1
(iii) -1 (iv) 10

3 A function calls itself directly or indirectly is called ____________.
(i) type conversion (ii) repetition
(iii) recursion (iv) union

4 ____________ is a collection of different data types.
(i) Array (ii) Files
(iii) String (iv) Structure

5 ____________ mode argument is used to truncate.
(i) a (ii) b (iii) f (iv) w

SECTION - B (15 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 3 = 15)

6 a Explain data types in C programming.
OR
b Write a note on arrays supported in C programming.

7 a Explicate branching statements in C with example.
OR
b Write short notes on gets and puts.

8 a Give a brief note on passing arrays to functions.
OR
b What are the storage classes? Explain with example.

9 a How to read and write a string in C? Explain.
OR
b What is meant by pointers? Explain.

Cont...

10 a Elucidate about bitwise operations.

OR

b Write short note on register variables.

SECTION -C (30 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 6 = 30)

11 a Elucidate types of operators supported in C with relevant examples.

OR

b Write C program to find the sum of n numbers.

12 a Explain for.... loop, while...loop and do...while loop in C with relevant example.

OR

b Explicate the purpose of break and continue statements with relevant C program.

13 a Define functions. Explain passing arguments to function.

OR

b Briefly explain multi-dimensional arrays with example.

14 a Discuss any six string functions with suitable example.

OR

b How structures differ from unions in C programming?

15 a Describe about files and its types in supported in C.

OR

b How to open and close a file in C program? Explain with relevant C program.

Z-Z-Z END

18MCU09

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2022
(Third Semester)

Branch - **MATHEMATICS WITH COMPUTER APPLICATIONS**

PARTIAL DIFFERENTIAL EQUATIONS & FOURIER SERIES

Time: Three Hours | Maximum: 50 Marks

SECTION-A (5 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 1 = 5)

1 Any solution of a PDE of the first order which contains two arbitrary constants is ________.
(i) singular solution (ii) complete solution
(iii) general solution (iv) particular solution

2 The second order equations is elliptic if ________.
(i) $S^2 - 4RT = 0$ (ii) $S^2 - 4RT > 0$
(iii) $S^2 - 4RT < 0$ (iv) $S^2 - 4RT \geq 0$

3 Linear PDE $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ is called as ________.
(i) one dimensional wave (ii) diffusion equation
(iii) harmonic equation (iv) one dimensional heat equation

4 The fundamental period for $\cos 2x$ is ______.
(i) 2π (ii) 0
(iii) $\frac{\pi}{2}$ (iv) π

5 Fourier cosine transform of $\sin ax^2$ $(a > 0)$ is ______.
(i) $\frac{1}{2a}\cos\left(\frac{w^2}{4a} + \frac{\pi}{4}\right)$ (ii) $\frac{1}{2a}\sin\left(\frac{w^2}{4a} + \frac{\pi}{4}\right)$
(iii) $\frac{1}{\sqrt{2a}}\cos\left(\frac{w^2}{4a} + \frac{\pi}{4}\right)$ (iv) $\frac{1}{\sqrt{2a}}\cos\left(\frac{w^2}{4a} - \frac{\pi}{4}\right)$

SECTION - B (15 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 3 = 15)

6 a Eliminate the constants a and b from the equation $ax^2 + by^2 + z^2 = 1$.
OR
b Eliminate the arbitrary function f from the equation $z = x + y + f(xy)$.

7 a Solve the equation $\frac{\partial^4 z}{\partial x^4} + \frac{\partial^4 z}{\partial y^4} = 2\frac{\partial^4 z}{\partial x^2 \partial y^2}$.
OR
b Reduce the equation $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$ to canonical form.

8 a Describe the Fourier series for the period 2L.
OR
b Find the Fourier series for $f(x) = x + \pi$ if $-\pi < x < \pi$ and $f(x + 2\pi) = f(x)$.

Cont...

9 a Compute the total square error of F with $N=3$ relative to $f(x)=x+\pi \text{ if } -\pi<x<\pi$.

OR

b Find the Fourier Cosine transform of e^{-x}.

10 a Find the temperature in the infinite bar if the temperature is $f(x)=\begin{cases} U_0 = \text{cons tan } t & \text{if } |x|<1 \\ 0 & \text{if } |x|>1 \end{cases}$

OR

b Write the physical assumptions for the motion of a stretched elastic membrane.

SECTION -C (30 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 6 = 30)

11 a Find the general solution of the differential equation $x^2\frac{\partial z}{\partial x}+y^2\frac{\partial z}{\partial y}=(x+y)z$.

OR

b Find the surface which intersects the surfaces of the system $z(x+y)=c(3z+1)$ orthogonally and which passes through the circle $x^2+y^2=1, z=1$.

12 a Derive one dimensional diffusion equation.

OR

b Find the solution of the equation $\frac{\partial^2 z}{\partial x^2}-\frac{\partial^2 z}{\partial y^2}=(x-y)$.

13 a Find the Fourier series of the periodic function $u(t)=\begin{cases} 0 & in -L<t<0 \\ E\sin\omega t & in\, 0<t<L \end{cases}, L=\frac{\pi}{\omega}, p=2L=\frac{2\pi}{\omega}$.

OR

b Find the two half-range expansions of the function $f(x)=\begin{cases} \frac{2k}{L}x & \text{if } 0<x<\frac{L}{2} \\ \frac{2k}{L}(L-x) & \text{if } \frac{L}{2}<x<L \end{cases}$

14 a Solve the heat problem $f(x)=\begin{cases} U_0 = \text{cons tan } t & \text{if } |x|<1 \\ 0 & \text{if } |x|>1 \end{cases}$ by the method of convolution.

OR

b Find the vibrations of a rectangular membrane of sides $a=4\,ft$ and $b=2\,ft$ if the tension is 12.5 Ib/ft, the density is 2.5 slugs/ft^2, the initial velocity is 0, and the initial displacement is $f(x,y)=0.1(4x-x^2)(2y-y^2)\,ft$.

15 a Find the Fourier cosine and sine integrals of $f(x)=e^{-kx}$.

OR

b Find the Fourier transform of $f(x)=e^{-ax^2}$, where $a>0$.

Z-Z-Z END

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2022
(Fifth Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

REAL ANALYSIS

Time: Three Hours | Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 x 1 = 10)

1 A function f is 1-1 if
(i) $f(x) = f(y) \Rightarrow x = y$ (ii) $x = y \Rightarrow f(x) = f(y)$
(iii) $f(x) = f(y) \Rightarrow x \neq y$ (iv) $f(x) = f(y)$

2 Compact subsets of metric spaces are------
(i) open (ii) closed (iii) countable (iv) compact

3 Every interval $[a, b] (a < b)$ is
(i) countable (ii) uncountable (iii) compact (iv) open

4 A metric space in which every Cauchy sequence converges is said to be----
(i) compact (ii) closed (iii) complete (iv) countable

5 $||x| - |y|| \leq ----$
(i) $|x - y|$ (ii) $|x + y|$ (iii) $|x| - |y|$ (iv) $|xy|$

6 $f(x) = \begin{cases} x & (x \text{ is rational}) \\ 0 & (x \text{ is irrational}) \end{cases}$ then f is continuous at $x =$
(i) 0 (ii) 1 (iii) $x = \infty$ (iv) $x = -\infty$

7 If f is differentiable on $[a, b]$, then f' cannot have any simple discontinuous on
(i) $[a, b)$ (ii) $[a, b]$ (iii) (a, b) (iv) $(a, b]$

8 $f(x) = e^{ix}$ then $|f'(x)| =$-----
(i) 1 (ii) 0 (iii) e^x (iv) e^{-x}

9 $\int_{-a}^{b} f dx =$--------.
(i) $inf\ L(P, f)$ (ii) $sup\ L(P, f)$ (iii) $m(b - a)$ (iv) $M(b - a)$

10 If f is a real, continuously differentiable function on $[a, b], f(a) = f(b) = 0$ then $\int_a^b x f(x) f'(x) dx =$------
(i) $\frac{1}{2}$ (ii) $-\frac{1}{2}$ (iii) 2 (iv) 1

SECTION - B (25 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** Marks (5 x 5 = 25)

11 a Show that every subset of a countable set A is countable.
(or)
b Prove that compact subsets of metric spaces are closed.

12 a If $\{I_n\}$ is a sequence of intervals in R^1 such that $I_n \supset I_{n+1} (n = 1,2, \ldots\ldots)$, then bring out $\bigcap_1^\infty I_n$ is not empty.
(or)
b State and prove Weierstrass theorem.

13 a Suppose f is a continuous mapping of a compact metric space X into a metric space Y. Then show that $f(x)$ is compact.
(or)

Cont...

b If f is continuous mapping of a metric space X into a metric space Y and if E is a connected subset of X then show that f(E) is connected.

14 a If $f(x) = \begin{cases} x^2 \sin\frac{1}{x}, & (x \neq 0) \\ 0, & (x = 0) \end{cases}$, then show that f is differentiable at all points of x.

(or)

b Verify $f(x) = x$ and $g(x) = x + x^2 e^{\frac{t}{x^2}}$ satisfies mean value theorem.

15 a Prove that $\int_{-a}^{b} f dx \leq \int_{a}^{-b} f dx$.

(or)

b State and prove fundamental theorem of calculus.

SECTION -C (40 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 8 = 40)

16 a Let $\{E_n\}, n = 1,2,3 \cdots\cdots$ be a sequence of countable sets and $S = \bigcup_{n=1}^{\infty} E_n$ then show that S is countable.

OR

b Prove that a subset E of Y is open relative to Y if and only if $E = Y \cap G$ for some open subset G of X and $Y \subset X$.

17 a Let P be a nonempty perfect set in R^k. Then prove that P is uncountable.

OR

b Prove that (i) In any metric space X, every convergent sequence is a Cauchy sequence. (ii) In R^k every Cauchy sequence converges.

18 a Prove that a mapping f of a metric space X into a metric space Y is continuous on X if and only if $f^{-1}(V)$ is open in X for every open set V in Y.

OR

b Prove that If f is a continuous mapping of a metric space X into a metric space Y and if E is a connected subset of X then $f(E)$ is connected.

19 a Prove that If f and g are continuous real functions on $[a, b]$ which are differentiable in (a, b) then there is point $x \in (a, b)$ at which $[f(b) - f(a)]g'(x) = [g(b) - g(a)]f'(x)$.

OR

b State and Prove Taylor's theorem.

20 a Show that $f \in \Re(\alpha)$ on $[a, b]$ if and only if for every $\in > 0$ there exists a partition P such that $(P, f, \alpha) - L(P, f, \alpha) < \in$.

OR

b Prove that If f maps $[a, b]$ into R^k and if $f \in \Re(\alpha)$ for some monotonically increasing function α on $[a, b]$ then $|f| \in \Re(\alpha)$ and $\left|\int_a^b f\, d\alpha\right| \leq \int_a^b |f|\, d\alpha$.

Z-Z-Z END

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2022
(Fifth Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

LINEAR ALGEBRA

Time: Three Hours Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 x 1 = 10)

1. If v is a vector space, then the elements of v and the elements of F are called ______.
   (i) Basis and dimensions (ii) Linear Combination and space
   (iii) Vectors and Scalars (iv) Linearly dependent and linearly independent

2. If $S = \{v_1, v_2, ..., v_n\}$, a set of vectors is a finite dimensional vector space v, then S is called a basis for v if ....
   (i) S *spans* v (ii) S is linearly independent
   (iii) either (i) *or* (ii) (iv) both (i) *and* (ii)

3. What is the dimension of zero vector space?
   (i) Not defined(ii) 1 (iii) 0 (iv) infinite

4. Let $S = \{(-1,0,1), (2,1,4)\}$. What is the value of k for which the vector $(3k + 2,3,10)$ belongs to the linear span of S?
   (i)8 (ii) 4 (iii) -2 (iv) 2

5. Identify $(\alpha u + \beta\, v)(T_1T_2)$ is
   (i) $\alpha(u(T_1T_2)) + \beta(v(T_1T_2))(ii)\alpha(u(T_1T_2)) + \beta(u(T_1T_2))$
   (iii) $\alpha(u((T_2)) + \beta(v(T_1))$ (iv) $\alpha\, u(T_1T_2) + \beta v(T_1T_2)$

6. If $T \in A(v)$ then the range of T, vT is defined by ......
   $(i)\{vT/v \in V\}$ (ii) $\{T/v \in VT\}$ (iii) $\{vT/v \in T\}$ (iv) $\{tv/t \in T\}$

7. What is the maximum positive rank of an $m \times n$ matrix A that is not square?
   (i) rank $(A) \geq \min(m,n)$ (ii) rank $(A) \leq \min(m,n)$
   (iii) rank $(A) = \max(m,n)$ (iv) rank $(A) = \max(m,n)$

8. The transition matrices are .....
   (i) not at all invertible (ii) invertible always
   (iii) invertible sometimes (iv) data not complete

9. Find the wrong one from the following statements:
   (i) If A is diagonalizable and invertible, then A^{-1} is diagonalizable.
   (ii) If A is diagonalizable, then A^T is diagonalizable.
   (iii) If every eigenvalue of a matrix A has algebraic multiplicity 1, then A is diagonalizable.
   (iv) An $n \times n$ matrix with fewer than n distinct eigenvalues is not diagonalizable.

10. If 0 is an eigenvalue of a matrix A, then the set of columns of A is ________.
   (i) Linearly independent or linearly dependent
   (ii) Linearly dependent always
   (iii) Linearly independent always
   (iv) Cannot be determined

SECTION - B (25 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** Marks (5 x 5 = 25)

11. a) If V is the internal direct sum of $U_1, ..., U_n$, then show that V is isomorphic to the external direct sum of $U_1, ..., U_n$.
   (OR)
   b) Show that $L(s)$ is a subspace of V.

Cont...

12. a) Show that $A(A(W)) = W$.

(OR)

b) Describe about inner product space with an example.

13. a) If V is finite dimensional over F, then analyze $T \in A(V)$ is invertible if and only if the constant term of the minimal polynomial for T is not 0.

(OR)

b) If $\lambda \in F$ is a characteristic root of $T \in A(V)$, then show that for any polynomial $q(x) \in F[x]$, $q(x)$ is a characteristic root of $q(T)$.

14. a) Let $A = \begin{bmatrix} 1 & -3 & -4 \\ -4 & 6 & -2 \\ -3 & 7 & 6 \end{bmatrix}$ and $b = \begin{bmatrix} 3 \\ 3 \\ -4 \end{bmatrix}$. Calculate whether b is the column space of A.

(OR)

b) If $A = \begin{bmatrix} 6 & 1 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}$. Verify $\det AB = (\det A)(\det B)$.

15. a) Let $A = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}$, $u = \begin{bmatrix} 6 \\ -5 \end{bmatrix}$, $v = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$. State whether u *and* v are eigenvalue of A.

(OR)

b) Bring out the eigenvalue of $A = \begin{bmatrix} 2 & 3 \\ 3 & -6 \end{bmatrix}$.

SECTION -C (40 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 8 = 40)

16. a) If $V_1, V_2, \dots V_n$ is a basis of V over F and if $w_1, w_2, \dots, w_n$ in V are linearly independent over F, then justify $m \leq n$.

(OR)

b) If V is finite dimensional and if W is a subspace of V, classify W is finite dimensional $\dim W \leq \dim V$ and $\dim \frac{V}{W} = \dim V - \dim W$.

17. a) If $u, v \in V$, then show that $|(u, v)| \leq \|u\| \|v\|$.

(OR)

b) If V is a finite dimensional inner product space and if W is a subspace of V, then justify $V = W + W^{\perp}$. More particularly, prove that V is the direct sum of W *and* $W^{\perp}$.

18. a) If A is an algebra, with unit element over F, then analyze A is isomorphic to a sub algebra of $A(V)$ for some vector space V over F.

(OR)

b) If $\lambda \in F$ is a characteristic root of $T \in A(V)$, show that λ is a root of the minimal polynomial of T. In particular, prove that T only has a finite number of characteristic roots in F.

19. a) Calculate the inverse of the matrix $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 4 & -3 & 8 \end{bmatrix}$, if it exists.

(OR)

b) Discover det A, *where* $A = \begin{bmatrix} 0 & 1 & 2 & -1 \\ 2 & 5 & -7 & 3 \\ 0 & 3 & 6 & 2 \\ -2 & -5 & 4 & -2 \end{bmatrix}$.

20. a) Discover the characteristic equation of $A = \begin{bmatrix} 5 & -2 & 6 & -1 \\ 0 & 3 & -8 & 0 \\ 0 & 0 & 5 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

(OR)

b) Let $A = \begin{bmatrix} 7 & 2 \\ -4 & 1 \end{bmatrix}$. Discover a formula for A^k, given that $A = PDP^{-1}$, where $P = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}$ and $D = \begin{bmatrix} 5 & 0 \\ 0 & 3 \end{bmatrix}$.

Z-Z-Z END

18MCU24

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2022
(Fifth Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

ADVANCED DIFFERENTIAL EQUATIONS

Time: Three Hours Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 x 1 = 10)

1. Let $A = \begin{bmatrix} 2 & -3 \\ 4 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -4 \\ 5 & 1 \end{bmatrix}$. Find $3A - 2B$.
   i) $\begin{bmatrix} -1 & 0 \\ 19 & 2 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & -1 \\ 2 & -19 \end{bmatrix}$ (iii) $\begin{bmatrix} 0 & 1 \\ -2 & 19 \end{bmatrix}$ (iv) $\begin{bmatrix} 0 & -1 \\ 2 & 19 \end{bmatrix}$
2. Find the eigenvalues of $A = \begin{bmatrix} 0 & 1 \\ -6 & 5 \end{bmatrix}$.
   (i) 2 *and* 3 (ii) -2 *and* 3 (iii) -2 *and* -3 (iv) 2 *and* -3
3. What is the value of e^x?
   (i) $\sum_{n=0}^{\infty} x^n$ (ii) $\sum_{n=0}^{\infty} \frac{x^n}{2n!}$ (iii) $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ (iv) $\sum_{n=0}^{\infty} \frac{(-1)x^{-1}}{n!}$
4. Find the value of $\lim_{n\to\infty} \frac{(n-1)!}{n!}$.
   $i)1$ (ii)-1 (iii) ∞ (iv) 0
5. What is the general solution of Bessel's equation of integral order n?
   i) $y(x) = x^2[C_1 J_p(kx^\beta) + C_2 J_{-p}(kx^\beta)]$ (ii) $y(x) = C_1 J_n(x) + C_2 Y_n(x)$
   $(iii) y(x) = C_1 J_{n-1} x + C_2 Y_n(x)$ (iv) $y(x) = C_1 J_{-p}(x) + C_2 Y_{n-1}(x)$
6. What is the value of $\Gamma\left(\frac{1}{2}\right)$?
   i) π (ii)$-\pi/2$ (iii) $\sqrt{\pi}/2$ (iv) $\sqrt{\pi}$
7. Which point is called isolated if some neighbourhood of it contains no other such point?
   i) Singular point (ii) Regular point
   (iii) Critical point (iv) Stable spiral point
8. What is the characteristic equation of the Jacobian matrix?
   i)$\lambda^2 - ab = 0$ (ii) $\lambda^2 + ab = 0$ (iii) $\lambda^2 + 2ab = 0$ (iv) $\lambda^2 = 0$
9. What is the complete integral of the equations $(p+q)(z - xp - yq) = 1$?
   i) $ax + by + \frac{1}{a+b}$ (ii) $xp + yq + \frac{1}{p+q}$
   (iii) $\frac{x}{p} + \frac{y}{q} + \frac{1}{p+q}$ (iv)$(p+q) + xp + yq$
10. What is the complete integral of $f = xpq + yp^2 - 1 = 0$ obtained by Charpit's method?
    i) $(z-b)^2 = 4(ax - y)$ (ii) $(z-b)^2 = 4(ax + y)$
    (iii) $(z+b)^2 = 4(ax+y)$ (iv) $(z+b)^2 = 4ax + y^2$

SECTION - B (25 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** Marks (5 x 5 = 25)

11. a) Let $A(t) = \begin{bmatrix} t & 2t-1 \\ t^3 & 1/t \end{bmatrix}$ and $B(t) = \begin{bmatrix} 1-t & 1+t \\ 3t^2 & 4t^3 \end{bmatrix}$. Show that the product law for differentiation, $(AB)' = A'B + AB'$.

    (OR)

Cont...

b) Bring out a general solution of the system $x' = \begin{bmatrix} 1 & -3 \\ 3 & 7 \end{bmatrix} x$.

12. a) Solve the equation $x^2y' = y - x - 1$.

(OR)

b) Analyze the nature of the point $x = 0$ for the differential equation $x^4y'' + (x^2 \sin x)y' + (1 - \cos x)y = 0$.

13. a) Solve the Airy equation $y'' + qxy = 0$.

(OR)

b) Produce the general solution of the differential equation $xy'' + 3y' + xy = 0$ in terms of Bessel functions.

14. a) Bring out the critical points of the system $\frac{dx}{dt} = 14x - 2x^2 - xy, \frac{dy}{dt} = 16y - 2y^2 - xy$.

(OR)

b) Determine the type and stability of the critical point (4,3)of the almost linear systems $\frac{dx}{dt} = 33 - 10x - 3y + x^2, \frac{dy}{dt} = -18 + 6x + 2y - xy$.

15. a) Show that the equations $xp = yq, z(xp + yq) = 2xy$ are compatible and solve them.

(OR)

b) Find a complete integral of the equation $P^2x + q^2y = z$.

SECTION -C (40 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 8 = 40)

16. a) Bring out a general solution of the system $x_1' = 4x_1 + 2x_2, x_2' = 3x_1 - x_2$.

(OR)

b) Bring out a general solution of the system $x' = \begin{bmatrix} 9 & 4 & 0 \\ -6 & -1 & 0 \\ 6 & 4 & 3 \end{bmatrix} x$.

17. a) Solve the equation $y' + 2y = 0$.

(OR)

b) Bring out the general solution in powers of x of $(x^2 - 4)y'' + 3xy' + y = 0$. Then find the particular solution with $y(0) = 4, y'(0) = 1$.

18. a) Calculate whether or not the equation $x^2y'' - xy' + (x^2 - 8)y = 0$ has two linearly independent Frobenius series solutions.

(OR)

b) Solve the equation $4x^2y'' + 8xy' + (x^4 - 3)y = 0$.

19. a) Examine that $(0,0)$ is the only critical point of the system $\frac{dx}{dt} = -ky + x(1 - x^2 - y^2)$

$\frac{dy}{dt} = kx + y(1 - x^2 - y^2)$

(OR)

b) Show that the linearization of $\frac{dx}{dt} = 5x - x^2 - xy, \frac{dy}{dt} = -2y + xy$ at $(5,0)$ is $u' = -5u - 5v, v' = 3v$. Then show that the co-efficient matrix of this linear system has the negative eigenvalue $\lambda_1 = -5$ and the positive eigenvalue $\lambda_2 = 3$. Hence (5,0) is a saddle point for the above system.

20. a) Show that the equations $xp - yq = x,\ x^2p + q = xz$ are compatible and obtain their solution.

(OR)

b) Find a complete integral of the equations $p^2y(1 + x^2) = qx^2$.

Z-Z-Z END

TOTAL PAGES: 2
18MCU25

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2022
(Fifth Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

DATABASE MANAGEMENT SYSTEMS

Time: Three Hours Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 x 1 = 10)

1. A Relational Database consist of a collection of ________________
   (i)Table (ii) Field (iii) Record (iv) Keys.
2. A ____________ in a table represents a relationship among a set of values.
   (i) Column (ii) Key (iii)Row (iv)Entry
3. Which language can user request information from a Database?
   (i) Query (ii) Relational (iii) Structural (iv) Compiler
4. OLAP stands for ___________
   (i) online analytical processing (ii) online analyzing processing
   (iii) online transaction processing (iv) online aggregate processing
5. The Most commonly used operation in relational algebra is __________
   (i) Storage cell (ii) buffer cell (iii) garbage cell (iv) Register Cell
6. Which are joint types on join condition ______________
   (i) Cross join (ii) Nature join
   (iii) Join with USING clause (iv) All the Above
7. ________of the following makes the transaction permanent in the Database
   (i) View (ii) Commit (iii) Rollback (iv) Flashback
8. ________________ is used to remove the privileges from the user X
   (i) Remove update on department from userX
   (ii) Revoke update on employee from userX
   (iii) Delete select on department from userX
   (iv) Grand update on employee from userX
9. The Components of a ER Model has _____________
   (i) Entity (ii) Attribute (iii) Relationship (iv) All the Above
10. Which of the following symbols represent entity sets in an ER diagram_____
    (i) Divided rectangles (ii) Diamonds
    (iii) Lines (iv) Undivided rectangles

SECTION - B (25 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** Marks (5 x 5 = 25)

11. a) What do you mean by view of Data in SQL.

(OR)

b) What are the Data – Manipulated Language?

12. a) Listout the Basic Types in SQL Standards.

(OR)

b) Explain about the Basic Structure of SQL Queries.

Cont...

13. a). Discuss about the Aggregation with Grouping in SQL.

(OR)

b) Describe the concept of Scalar Sub queries.

14. a) Write short notes on the Design Phase.

(OR)

b) Explain about the Large – Object Types in SQL.

15. a) Write about the representation of Relationship Set with Suitable Examples.

(OR)

b) What is Design Phase in SQL?

SECTION – C (40 marks)

Answer **ALL** Questions

All Questions carry **EQUAL** Marks (5 X 8 = 40 marks)

16. a) Discuss about the Data – Definition Language in detail.

(OR)

b) Write about Database Architecture in Detail.

17. a) Explain the Natural Join in Basic Structure in SQL.

(OR)

b) Elaborate about the Null Values in SQL with Sample Queries.

18. a) Discuss about the Join Expressions and its Conditionals in detail.

(OR)

b) Elaborate about the Integrity Constraints and Relations in SQL.

19. a) Explain about the Roles and revoking of Privileges in SQL.

(OR)

b) Elaborate about the Embedded SQL.

20. a) Briefly explain about the Entity Set in E- R Model.

(OR)

b) Explain the concept of Mapping Coordinates in Constraints.

Z-Z-Z END