22MCU101

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2022
(First Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

CALCULUS

Time: Three Hours　　　　Maximum: 50 Marks

SECTION-A (5 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** marks　　(5 x 1 = 5)

1 The distance from the point $P(2,-1,7)$ to the point $Q(1,-3,5)$ is

(i) $\sqrt{15}$　　(ii) $\sqrt{10}$
(iii) 3　　(iv) 9

2 $\lim_{(x,y)\to(1,2)}(x^2y^3 - x^3y^2 + 3x + 2y) =$ ______.

(i) 10　　(ii) 11
(iii) 9　　(iv) 12

3 If $f(x,y,z) = x\sin yz$ then $grad(f) =$ _____.

(i) $(\sin yz, xz\sin xz, xy\sin yz)$　　(ii) $(\cos yz, xz\cos yz, xy\cos yz)$
(iii) $(\sin yz, xz\cos yz, xy\cos yz)$　　(iv) $(\sin yz, xz, xz\sin yz, xy\cos yz)$

4 $\int_0^2\int_1^2 (x-3y^2)dydx =$ ____________.

(i) -12　　(ii) 12
(iii) 21　　(iv) -21

5 If $x = u^2 - v^2$, $y = 2uv$ then $\frac{\partial(x,y)}{\partial(u,v)} =$ ______.

(i) $u^2 + v^2$　　(ii) $4uv$
(iii) $4u^2 - 4v^2$　　(iv) $4u^2 + 4v^2$

SECTION - B (15 Marks)

Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks　　(5 x 3 = 15)

6a) Find the angle between the vectors $a = (2,2,-1)$ *and* $b = (5,-3,2)$.

OR

b) Find the length of the arc of the circular helix with vector equation $r(t) = \cos t\ \vec{i}\_\sin t\ \vec{j} + t\ \vec{k}$ from the point $(1,0,0)$ to the point $(1,0,2\pi)$.

7a) Find the level curves of the function $f(x,y) = 6 - 3x - 2y$ for the values $k = -6,0,6,12$.

OR

b) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if z is defined implicitly as a function of x and y by the equation $x^3 + y^3 + z^3 + 6xyz = 1$.

8a) If $z = e^x \sin y$ where $x = st^2$ and $y = s^2t$ find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$.

OR

b) Find the direction derivative $D_u f(x,y)$ if $f(x,y) = x^3 - 3xy + 4y^2$ and u is the unit vector given by angle $\theta = \frac{\pi}{6}$ what is $D_u f(1,2)$.

9a) Use the midpoint rule with $m = n = 2$ to estimate the value of the integral $\iint_R (x-3y^2)dA$ where $R = \{(x,y)/0 \le x \le 2,,1 \le y \le 2\}$.

OR

b) Evaluate $\iint_D (x+2y)\, dA$ where D is the region bounded by the parabolas $y = 2x^2$ and $y = 1 + x^2$.

10a) Evaluate the triple integral $\iiint_B xyz^2\, dV$, where B is the rectangular box $B = \{(x, y, z) / 0 \le x \le 1,\ -1 \le y \le 2,\ 0 \le z \le 3$.

OR

b) The point $(2, \frac{\pi}{4}, \frac{\pi}{3})$ is given in spherical coordinates , plot the point and find its rectangular coordinates.

SECTION -C (30 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 6 = 30)

11a Find a vector perpendicular to the plane that passes through the points $P(1,4,6)$, $Q(-2,5,-1)$, $R(1,-1,1)$.

OR

b) Find the curvature of the twisted cubic $r(t) = < t, t^2, t^3 >$ at a general point and at $(0,0,0)$.

12a) (i) Find $\lim_{(x,y)\to(0,0)} \frac{3x^2 y}{x^2 + y^2}$ if it exists .

(ii) If $f(x, y) = \frac{xy^2}{x^2 + y^2}$, then does $\lim_{(x,y)\to(0,0)} f(x, y)$ exist?

OR

b) (i) Find the second Partial derivatives of $f(x, y) = x^3 + x^2y^3 - 2y^2$.

(ii) Calculate f_{yz} if $f(x, y, z) = \sin(3x + yz)$.

13a)(i) If $g(s,t) = f(s^2 - t^2, t^2 - s^2)$ and f is differentiable show that g satisfies the equation $t\frac{\partial g}{\partial s} + S\frac{\partial g}{\partial t} = 0$.

(ii) If $z = f(x, y)$ has continuous second order partial derivatives and $x = t^2 + s^2$ and $y = 2rs$ then find $\frac{\partial z}{\partial r}$ and $\frac{\partial^2 z}{\partial r^2}$.

OR

b) Find the local maximum and minimum values and saddle points of $f(x, y) = x^4 + y^4 + 4xy + 1$.

14a) Evaluate $\iint_R (3x + 4y^2)\, dA$ where D is the region in the upper half-plane bounded by the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

OR

b) Evaluate the mass and centre of mass of a triangular lamina with vertices $(0,0), (1,0)$ and $(0,2)$ if the density function is $\rho(x, y) = 1 + 3x + y$.

15a) Find center and mass of the solid of constant density that is bounded by the parabolic cylinder $x = y^2$ and the planes $x = z, z = 0$ and $x = 1$.

OR

b) (i) Plot the point with cylindrical coordinates $(2, \frac{2\pi}{3}, 1)$ and find its rectangular coordinates.

(ii) Find cylindrical coordinates of the point with rectangular coordinates . $(3,-3,-7)$.

Z-Z-Z END