19MCU05

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2022
(Second Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

ANALYTICAL GEOMETRY OF 3D AND VECTOR CALCULUS

Time: Three Hours Maximum: 50 Marks

SECTION-A (5 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 1 = 5)

1. The length of the perpendicular from the point (4,3,0) to the plane $2x + 3y + \sqrt{3}z + 3 = 0$ is ________
   (i) $\frac{1}{4}$ (ii) 5 (iii) $\frac{3}{4}$ (iv) $\frac{1}{5}$

2. The equation of the tangent plane at (0,0,1) to the sphere $x^2 + y^2 + z^2 = 1$ is ___
   (i) $x = 0$ (ii) $y = 0$ (iii) $z = 1$ (iv) $x + y + z = 1$

3. A cone of second degree can be found to pass through ______concurrent lines.
   (i) 5 (ii) 6 (iii) 4 (iv) 3

4. If C is the straight line joining (0,0,0) and (1,1,1) then $\int_C \boldsymbol{r}.\boldsymbol{dr}$ is _______
   (i) $\frac{1}{2}$ (ii) 1 (iii) $\frac{3}{2}$ (iv) 2

5. Gauss' divergence theorem connects________.
   (i) Line integral and double integral (ii) Line integral and surface integral
   (iii) Double integral and surface integral (iv) Surface integral and volume integral

SECTION – B (15 Marks)

Answer **ALL** Questions
ALL questions carry **EQUAL** marks (5 x 3 = 15)

6. a) Find the distance between the parallel planes $x + 2y - 3z + 1 = 0$ and $2x + 4y - 4z + 5 = 0$.

   OR

   b) Find the perpendicular distance of $P(1,2,3)$ from the line $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$.

7. a) Find the radius and centre of the sphere $x^2 + y^2 + z^2 - 2x + 4y - 6z = 2$.

   OR

   b) Find the equation of the sphere through the circle $x^2 + y^2 + z^2 = 9, 2x + 3y + 4z = 5$ and the point (1,2,3).

8. a) Find the equation of the cone whose vertex is at the origin and which passes through the curve given by the equations $ax^2 + by^2 + cz^2 = 1, lx + my + nz = p$.

   OR

   b) Find the equation to the right circular cone whose vertex is $P(2, -3,5)$, axis PQ which makes equal angles with the axes and semi-vertical angle is 30°.

9. a) Evaluate $\int_C (2 + x^2y)\, ds$, where C is the upper half of the unit Circle $x^2 + y^2 = 1$.

   OR

   b) Evaluate $\int_C x^4dx + xydy$, where C is the triangular curve consisting of the line segments from (0,0) to (1,0), from (1,0) to (0,1), and from (0,1) to (0,0).

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10. a) Find a parametric representation for the cylinder $x^2 + y^2 = 4 \ \ 0 \le z \le 1$.

OR

b) Compute the surface integral $\iint_S x^2 \, dS$, where S is the unit sphere $x^2 + y^2 + z^2 = 1$.

SECTION -C (30 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** Marks (5 x 6 = 30)

11. a) Find the volume of a tetrahedron in terms of the lengths of the three edges which meet in a point and of the angles which these edges make with each other in pairs.

OR

b) Show that the planes $ax + hy + gz = 0, hx + by + fz = 0, gx + fy + cz = 0$ have a common line of intersection if $\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$ and the direction ratios of the line satisfy the equations $\frac{l^2}{\partial\Delta/\partial a} = \frac{m^2}{\partial\Delta/\partial b} = \frac{n^2}{\partial\Delta/\partial c}$.

12. a) Find the equation of the circle circumscribing the triangle formed by the three points $(a, 0,0), (0, b, 0), (0,0, c)$. Also Obtain the co-ordinates of the centre of this circle.

OR

b) Show that the spheres $x^2 + y^2 + z^2 = 64$ and $x^2 + y^2 + z^2 - 12x + 4y - 6z + 48 = 0$ touch internally and find their point of contact.

13. a) Two cones pass through the curves $y = 0, z^2 = 4ax; x = 0, z^2 = 4$ and they have a common vertex. The plane $z = 0$ meets them in two conics that intersect in four concyclic points. Show that the vertex lies on the surface $z^2 \left(\frac{x}{a} + \frac{y}{b}\right) = 4(x^2 + y^2)$.

OR

b) Find the equation to the lines in which the planes $2x + y - z = 0$ cuts the cone $4x^2 - y^2 + 3z^2 = 0$.

14. a) (i) If $\boldsymbol{F}(x, y) = (3 + 2xy)\boldsymbol{i} + (x^2 - 3y^2)\boldsymbol{j}$, find a function f such that $\boldsymbol{F} = \nabla f$.
(ii) Evaluate the line integral $\int_C \boldsymbol{F}.d\boldsymbol{r}$, where C is the curve given by $\boldsymbol{r}(t) = e^t \sin t \, \boldsymbol{i} + e^t \cos t \, \boldsymbol{j} \ \ 0 \le t \le \pi$.

OR

b) (i) Show that $\boldsymbol{F}(x, y, z) = y^2z^3\boldsymbol{i} + 2xyz^3\boldsymbol{j} + 3xy^2z^2\boldsymbol{k}$ is a conservative vector field.
(ii) Find a function f such that $\boldsymbol{F} = \nabla f$.

15. a) Find the tangent plane to the surface with parametric equations $x = u^2, y = v^2, z = u + 2v$ at the point (1,1,3).

OR

b) Evaluate $\iint_S \boldsymbol{F}.d\boldsymbol{S}$, where $\boldsymbol{F}(x, y, z) = y\boldsymbol{i} + x\boldsymbol{j} + z\boldsymbol{k}$ and S is the boundary of the solid region E enclosed by the paraboloid $z = 1 - x^2 - y^2$ and the plane $z = 0$.

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