18MCU24

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2022
(Fifth Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

ADVANCED DIFFERENTIAL EQUATIONS

Time: Three Hours Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 x 1 = 10)

1. Let $A = \begin{bmatrix} 2 & -3 \\ 4 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -4 \\ 5 & 1 \end{bmatrix}$. Find $3A - 2B$.
   i) $\begin{bmatrix} -1 & 0 \\ 19 & 2 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & -1 \\ 2 & -19 \end{bmatrix}$ (iii) $\begin{bmatrix} 0 & 1 \\ -2 & 19 \end{bmatrix}$ (iv) $\begin{bmatrix} 0 & -1 \\ 2 & 19 \end{bmatrix}$
2. Find the eigenvalues of $A = \begin{bmatrix} 0 & 1 \\ -6 & 5 \end{bmatrix}$.
   (i) 2 and 3 (ii) -2 and 3 (iii) -2 and -3 (iv) 2 and -3
3. What is the value of e^x?
   (i) $\sum_{n=0}^{\infty} x^n$ (ii) $\sum_{n=0}^{\infty} \frac{x^n}{2n!}$ (iii) $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ (iv) $\sum_{n=0}^{\infty} \frac{(-1)x^{-1}}{n!}$
4. Find the value of $\lim_{n\to\infty} \frac{(n-1)!}{n!}$.
   $i)1$ (ii)-1 (iii) ∞ (iv) 0
5. What is the general solution of Bessel's equation of integral order n?
   i) $y(x) = x^2[C_1 J_p(kx^\beta) + C_2 J_{-p}(kx^\beta)]$ (ii) $y(x) = C_1 J_n(x) + C_2 Y_n(x)$
   $(iii) y(x) = C_1 J_{n-1} x + C_2 Y_n(x)$ (iv) $y(x) = C_1 J_{-p}(x) + C_2 Y_{n-1}(x)$
6. What is the value of $\Gamma\left(\frac{1}{2}\right)$?
   i) π (ii)$-\pi/2$ (iii) $\sqrt{\pi}/2$ (iv) $\sqrt{\pi}$
7. Which point is called isolated if some neighbourhood of it contains no other such point?
   i) Singular point (ii) Regular point
   (iii) Critical point (iv) Stable spiral point
8. What is the characteristic equation of the Jacobian matrix?
   i)$\lambda^2 - ab = 0$ (ii) $\lambda^2 + ab = 0$ (iii) $\lambda^2 + 2ab = 0$ (iv) $\lambda^2 = 0$
9. What is the complete integral of the equations $(p+q)(z - xp - yq) = 1$?
   i) $ax + by + \frac{1}{a+b}$ (ii) $xp + yq + \frac{1}{p+q}$
   (iii) $\frac{x}{p} + \frac{y}{q} + \frac{1}{p+q}$ (iv)$(p+q) + xp + yq$
10. What is the complete integral of $f = xpq + yp^2 - 1 = 0$ obtained by Charpit's method?
    i) $(z-b)^2 = 4(ax - y)$ (ii) $(z-b)^2 = 4(ax + y)$
    (iii) $(z+b)^2 = 4(ax + y)$ (iv) $(z+b)^2 = 4ax + y^2$

SECTION - B (25 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** Marks (5 x 5 = 25)

11. a) Let $A(t) = \begin{bmatrix} t & 2t-1 \\ t^3 & 1/t \end{bmatrix}$ and $B(t) = \begin{bmatrix} 1-t & 1+t \\ 3t^2 & 4t^3 \end{bmatrix}$. Show that the product law for differentiation, $(AB)' = A'B + AB'$.

(OR)

Cont...

b) Bring out a general solution of the system $x' = \begin{bmatrix} 1 & -3 \\ 3 & 7 \end{bmatrix} x$.

12. a) Solve the equation $x^2y' = y - x - 1$.

(OR)

b) Analyze the nature of the point $x = 0$ for the differential equation $x^4y'' + (x^2 \sin x)y' + (1 - \cos x)y = 0$.

13. a) Solve the Airy equation $y'' + qxy = 0$.

(OR)

b) Produce the general solution of the differential equation $xy'' + 3y' + xy = 0$ in terms of Bessel functions.

14. a) Bring out the critical points of the system $\frac{dx}{dt} = 14x - 2x^2 - xy, \frac{dy}{dt} = 16y - 2y^2 - xy$.

(OR)

b) Determine the type and stability of the critical point (4,3)of the almost linear systems $\frac{dx}{dt} = 33 - 10x - 3y + x^2, \frac{dy}{dt} = -18 + 6x + 2y - xy$.

15. a) Show that the equations $xp = yq, z(xp + yq) = 2xy$ are compatible and solve them.

(OR)

b) Find a complete integral of the equation $P^2x + q^2y = z$.

SECTION -C (40 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 8 = 40)

16. a) Bring out a general solution of the system $x_1' = 4x_1 + 2x_2, x_2' = 3x_1 - x_2$.

(OR)

b) Bring out a general solution of the system $x' = \begin{bmatrix} 9 & 4 & 0 \\ -6 & -1 & 0 \\ 6 & 4 & 3 \end{bmatrix} x$.

17. a) Solve the equation $y' + 2y = 0$.

(OR)

b) Bring out the general solution in powers of x of $(x^2 - 4)y'' + 3xy' + y = 0$. Then find the particular solution with $y(0) = 4, y'(0) = 1$.

18. a) Calculate whether or not the equation $x^2y'' - xy' + (x^2 - 8)y = 0$ has two linearly independent Frobenius series solutions.

(OR)

b) Solve the equation $4x^2y'' + 8xy' + (x^4 - 3)y = 0$.

19. a) Examine that (0,0) is the only critical point of the system $\frac{dx}{dt} = -ky + x(1 - x^2 - y^2)$
$\frac{dy}{dt} = kx + y(1 - x^2 - y^2)$

(OR)

b) Show that the linearization of $\frac{dx}{dt} = 5x - x^2 - xy, \frac{dy}{dt} = -2y + xy$ *at* (5,0) is $u' = -5u - 5v, v' = 3v$. Then show that the co-efficient matrix of this linear system has the negative eigenvalue $\lambda_1 = -5$ and the positive eigenvalue $\lambda_2 = 3$. Hence (5,0) is a saddle point for the above system.

20. a) Show that the equations $xp - yq = x,\ x^2p + q = xz$ are compatible and obtain their solution.

(OR)

b) Find a complete integral of the equations $p^2y(1 + x^2) = qx^2$.

Z-Z-Z END