22MAU101

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2022
(First Semester)

Branch – **MATHEMATICS**

CALCULUS - I

Time: Three Hours — Maximum: 50 Marks

SECTION-A (5 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 1 = 5)

1. Curve with vector function $\boldsymbol{r}(t) = < t, t^2, t^3 >$ is
   (i) twisted cubic (ii) cube
   (iii) toroidal spiral (iv) trefoilknot

2. If we study the functions of two variables by a table of values then it is
   (i) verbally (ii) numerically
   (iii) algebraically (iv) visually

3. Another name for critical point is
   (i) fixed point (ii) nodal point
   (iii) stationary point (iv) limit point

4. Which theorem gives practical method for evaluating double integral
   (i) Fubini's thorem (ii) Chain rule
   (iii) Change of variables (iv) Clairaut's thorem

5. Which coordinate system is useful in 3 dimensions
   (i) spherical (ii) rectangular
   (iii) Both (i) and (ii) (iv) Neither (i) or (ii)

SECTION - B (15 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 3 = 15)

6. (a) Find the domain of $\boldsymbol{r}$ for $\boldsymbol{r}(t) = < t^3, \ln(3-t), \sqrt{t} >$.

   (OR)

   (b) Show that if $|\boldsymbol{r}(t)| = c$, where c is a constant , then $\boldsymbol{r}'(t)$ is orthogonal to $\boldsymbol{r}(t)$for all t.

7. (a) Evaluate $f(3,2)$ and domain for $f(x, y) = \frac{\sqrt{x+y+1}}{x-1}$.

   (OR)

   (b) Show that $\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}$ does not exists.

8. (a) State Chain Rule for Case 1.

   (OR)

   (b) Write Second Derivative Test.

9. (a) Evaluate $\iint_R y \sin(xy)\, dA$, where $R = [1,2] \times [0, \pi]$.

   (OR)

   (b) Find the volume of the solid S that is bounded by the elliptic paraboloid $x^2 + y^2 + z = 16$, the planes $x = 2, y = 2$ and 3 coordinate plans.

10 (a) Find the rectangular coordinates for cylindrical coordinates $(2, \frac{2\pi}{3}, 1)$

   (OR)

   (b) The point $(2, \frac{\pi}{4}, \frac{\pi}{3})$ is given in Spherical coordinates. Find rectangular coordinates.

Cont...

SECTION -C (30 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** Marks (5 x 6 = 30)

11. (a) Find a vector function that represents the curve of intersection of the cyclinder $x^2 + y^2 = 1$ and the plane $y + z = 2$.

(OR)

(b) If $\boldsymbol{r}(t) = < f(t), g(t), h(t) > = f(t)\boldsymbol{i} + g(t)\boldsymbol{j} + h(t)\boldsymbol{k}$, where f, g, h are differentiable functions then prove that $\boldsymbol{r}'(t) = < f'(t), g'(t), h'(t) > = f'(t)\boldsymbol{i} + g'(t)\boldsymbol{j} + h'(t)\boldsymbol{k}$.

12. (a) Find the domain and range of $g(x,y) = \sqrt{9 - x^2 - y^2}$.

(OR)

(b) Sketch the graph of the function $f(x,y) = 6 - 3x - 2y$.

13. (a) Find $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$ if z is defined implicitly as a function of x,y by the equation $x^3 + y^3 + z^3$ +6xyz = 1.

(OR)

(b) Find the maximum value of the function $f(x,y,z) = x + 2y + 3z$ on the curve of intersection of the plane $x - y + z = 1$ and the cyclinder $x^2 + y^2 = 1$.

14. (a) Evaluate the iterated integrals $\int_0^3 \int_1^2 x^2 y \, dy \, dx$.

(OR)

(b) $\int \int_R (3x + 4y^2) \, dA$ where R is region in the upper half plane bounded by the circles $x^2 + y^2 = 1, x^2 + y^2 = 4$.

15. (a) Evaluate $\int \int \int_B e^{(x^2+y^2+z^2)^{3/2}} \, dv$, where B is the unit Ball $B = \{(x,y,z) | x^2 + y^2 + z^2 \leq 1\}$.

(OR)

(b) Use change of variables $x = u^2 - v^2, \; y = 2uv$ to evaluate the integral $\int \int_R y \, dA$ where R is the region bounded by x- axis and the parabolas $y^2 = 4 - 4x, and \; y^2 = 4 + 4x, y \geq 0$

Z-Z-Z END