22MAU102

**PSG COLLEGE OF ARTS & SCIENCE**
(AUTONOMOUS)

**BSc DEGREE EXAMINATION DECEMBER 2022**
(First Semester)

Branch – **MATHEMATICS**

**ANALYTICAL GEOMETRY OF 3D AND TRIGONOMETRY**

Time: Three Hours                                        Maximum: 50 Marks

**SECTION-A (5 Marks)**
Answer **ALL** questions
**ALL** questions carry **EQUAL** marks (5 x 1 = 5)

1. If the two lines are coplanar, they must ______
   (i) intersect (ii) surface (iii) ratio (iv) volume.

2. ______ is the locus of a point which moves in such a way that its distance from a fixed point is always constant.
   (i) square (ii) sphere (iii) rectangle (iv) cone

3. The cones $Ax^2 + By^2 + Cz^2 + 2Fyz + 2Gzx + 2Hxy = 0$ and $ax^2 + by^2 + cz^2 + 2fzy + 2gzx + 2hxy = 0$ are said to be
   (i) reciprocal (ii) polar plane (iii) Polar line (iv) vertex.

4. ______ is a surface generated by a straight line which is always parallel to a fixed line
   (i) cone (ii) cylinder (iii) polygon (iv) circle

5. $\cosh^2 x + \sinh^2 x$ = ____________.
   (i) cosx (ii) cosh2x (iii) sinhx (iv) tan x

**SECTION - B (15 Marks)**
Answer **ALL** Questions
**ALL** Questions Carry **EQUAL** Marks (5 x 3 = 15)

6. a) Prove that the lines $\frac{x+1}{-3} = \frac{y+10}{8} = \frac{z-1}{2}, \frac{x+3}{-4} = \frac{y+1}{7} = \frac{z-4}{1}$ are coplanar.

   **(or)**

   b) Find the symmetrical form of the equations of the line of intersection of the planes $x + 5y - z - 7 = 0$, $2x - 5y + 3z + 1 = 0$.

7. a) Find the equation of the sphere which has its centre at the point (6, -1, 2) and touches the plane $2x - y + 2z - 2 = 0$.

   **(or)**

   b) Show that the plane $2x - y - 2z = 16$ touches the sphere $x^2 + y^2 + z^2 - 4x + 2y + 2z - 3 = 0$ and find the point of contact.

8. a) Find the equation of the cone with vertex O and base curve, the conic in which the surface $ax^2 + by^2 + cz^2 = 1$ is cut by the plane $l_1x + m_1y + n_1z = p$.

   **(or)**

   b) Find the equations of the tangent planes to the cone $9x^2 - 4y^2 + 16z^2 = 0$ which contain the line $\frac{x}{32} = \frac{y}{72} = \frac{z}{72}$.

9. a) Find the equation of the cylinder whose generators are parallel to the z axis and the guiding curve is $ax^2 + by^2 = cz$, $lx + my + nz = p$.
   The equation of the z axis is $\frac{x}{0} = \frac{y}{0} = \frac{z}{1}$.

   **(or)**

**Cont...**

b) If OD is the diameter parallel to a secant APQ through A meeting the conicoid at P and Q show that $\frac{AP.AQ}{OD^2}$ is constant.

10. a) Express $\frac{\sin 6\theta}{\sin\theta}$ interms of cos θ.

**(or)**

b) If coshu = sec θ, show that $u = \log\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$.

## SECTION -C (30 Marks)

Answer **ALL** questions

**ALL** questions carry **EQUAL** Marks (5 x 6 = 30)

11. a) Find the shortest distance between the lines
$\frac{x-3}{-1}=\frac{y-4}{2}=\frac{z+2}{1};\frac{x-1}{1}=\frac{y+7}{3}=\frac{z+2}{2}$.

**(or)**

b) The Straight lines $\frac{x-\alpha}{l}=\frac{y-\beta}{m}=\frac{z-\gamma}{n};\frac{x-\alpha_1}{l_1}=\frac{y-\beta_1}{m_1}=\frac{z-\gamma_1}{n_1}$ are cut by a third whose direction cosines are λ, μ, ν. Show that the length intercepted on the third line is given by $\begin{vmatrix}\alpha-\alpha_1 & \beta-\beta_1 & \gamma-\gamma_1\\ l & m & n\\ l_1 & m_1 & n_1\end{vmatrix} \div \begin{vmatrix} l & m & n\\ l_1 & m_1 & n_1\\ \lambda & \mu & \nu\end{vmatrix}$ and deduce the length of the S.D.

12. a) Find the equation of the sphere which passes through the circle $x^2+y^2+z^2-2x-4y=0$, $x+2y+3z=8$ and touches the plane $4x+3y=25$.

**(or)**

b) A plane passes through a fixed point (a, b, c) and cuts the axes in A, B, C. Show that the locus of the centre of the sphere OABC is
$\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=2$.

13. a) Find the condition for the equation
$ax^2+by^2+cz^2+2fyz+2gzx+2fxy=0$ to represent a right circular cone. Obtain the equation of the axis and the vertical angle of the cone.

**(or)**

b) Find the condition for the equation

$$F(x, y, z) \equiv ax^2+by^2+cz^2+2fyz+2gzx+2hxy + 2ux+2vy+2wz+d=0$$

to represent a cone.

If it represents a cone, show that the co-ordinates of the vertex satisfy the equations $\frac{\partial F}{\partial x}=0, \frac{\partial F}{\partial y}=0, \frac{\partial F}{\partial z}=0, \frac{\partial F}{\partial t}=0$, where t is used to make F(x, y, z) homogeneous and is equated to unity after differentiation.

14. a) Find the equation of the right circular cylinder described on the circle through the points (a, 0, 0), (0, a, 0), (0, 0, a) as a guiding curve.

**(or)**

b) Find the locus of the point of intersection of three mutually perpendicular tangent planes to the central conicoid $ax^2+by^2+cz^2=1$.

15. a) Expand $\sin^3\theta\cos^5\theta$ in a series of sines of multiples of θ.

**(or)**

b) If tan (x + iy) = u + iv, prove that $\frac{u}{v}=\frac{\sin 2x}{\sinh 2y}$.

Z-Z-Z END