20MAU05

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2022
(Second Semester)

Branch – MATHEMATICS

ANALYTICAL GEOMETRY

Time: Three Hours Maximum: 50 Marks

SECTION-A (5 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (5x 1 = 5)

1. What is the semi-latus rectum of $\frac{10}{r} = 3\cos\theta + 4\sin\theta + 5$

   (i) 4 (ii) 1/2
   (iii) 5 (iv) 2

2 The value of λ for which the line $\frac{x-1}{1} = \frac{y-2}{\lambda} = \frac{z+1}{-1}$ and $\frac{x+1}{-\lambda} = \frac{y+1}{2} = \frac{z-2}{1}$ are perpendicular to each other is

   (i) -1 (ii) 1
   (iii) 0 (iv) 2

3 The equation $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ represents a real sphere if the radius is

   (i) equal to zero (ii) less than zero
   (iii) less than or equal to zero (iv) greater than zero

4 Two cones are said to be ---------- if one is the locus of the normal through the vertex to the tangent planes to the other

   (i) Right circular cone (ii) Reciprocal cones
   (iii) Double right circular cones (iv) Oblique cone

5 Which is parallel to the generator of the cylinder?

   (i) Axes (ii) Directrix
   (iii) Line (iv) Latus rectum

SECTION - B (15 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 3 = 15)

6. a. Trace the curve $\frac{10}{r} = 1 + 3\cos\theta + 4\sin\theta + 5$.

(OR)

b. A chord PQ of a conic subtends an angle of 2β of constant magnitude at the pole. Find the locus of the intersection of the tangents at P and Q.

7. a. Prove that the line $\frac{x-3}{2} = \frac{y-4}{3} = \frac{z-5}{4}$ is parallel to the plane $4x + 4y - 5z = 0$.

(OR)

b. Prove that the lines $\frac{x-5}{4} = \frac{y-7}{4} = \frac{z+3}{-5}$ and $\frac{x-8}{7} = \frac{y-4}{1} = \frac{z-5}{3}$ are coplanar.

8. a. Find the equation of the sphere which has its centre at the point (6, -1, 2) and touches the plane $2x - y + 2z - 2 = 0$.

(OR)

b. Show that the plane $2x - y - 2z = 16$ touches the sphere $x^2 + y^2 + z^2 - 4x + 2y + 2z - 3 = 0$ and find the point of contact.

Cont...

9. a. Find the equation of the right circular cone whose vertex is at the origin, whose axis is the line $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and which has a semi-vertical angle of 60°.

(OR)

b. Show that the equation of a right circular cone whose vertex is O, axis OZ and semi- vertical angle α is $x^2+y^2=z^2\tan^2\alpha$.

10. a. Find the equation of a right circular cylinder of radius 3 with axis $\frac{x+2}{3}=\frac{y-4}{6}=\frac{z-1}{2}$

(OR)

b. Find the equation of the cylinder whose generators are parallel to the line $\frac{x}{1}=\frac{y}{-2}=\frac{z}{3}$ and guiding curve $x^2+y^2=1,\ z=3$.

SECTION -C (30 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 6 = 30)

11. a. Find the equation of the chord of the conic $\frac{l}{r}=1+e\cos\theta$ joining the points whose vectorial angles are $\alpha-\beta$ and $\alpha+\beta$.

(OR)

b. If two conics have a common focus, show that two of their common chords pass through the point of intersection of their directrices.

12. a. Find the equation of the projection of the line $\frac{x-1}{2}=\frac{y+1}{-1}=\frac{z-3}{4}$ on the plane x+2y+z=6.

(OR)

b. Find the shortest distance between the lines $\frac{x-3}{-1}=\frac{y-4}{2}=\frac{z+2}{1}$ and $\frac{x-1}{1}=\frac{y+7}{3}=\frac{z+2}{2}$ and also find equation of line of shortest distance.

13. a. Find the equation of the sphere through the points (2, 3, 1), (5, -1, 2), (4, 3, -1), and (2, 5, 3)

(OR)

b. Find the equation of the sphere which passes through the circle $x^2+y^2+z^2-2x-4y=0,\ x+2y+3z=8$ and touches the plane $4x+3y=25$.

14. a. The plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ meets the co-ordinate axes in A, B, C. Prove that the equation to the cone generated by lines drawn from O to meet the circle ABC is $yz\left(\frac{b}{c}+\frac{c}{b}\right)+zx\left(\frac{c}{a}+\frac{a}{c}\right)+xy\left(\frac{a}{b}+\frac{b}{a}\right)=0$

(OR)

b. Prove that the general equation to a cone which touches the three co-ordinate planes is $\sqrt{fx}\pm\sqrt{gy}\pm\sqrt{hz}=0$

15. a. Find the equation of the cylinder whose generators are parallel to the z-axis and the guiding curve is $ax^2+by^2=cz,\ lx+my+nz=p$.

(OR)

b. Discuss the equation of the right circular cylinder described on the circle passing through the points (a, 0, 0), (0, a, 0), (0, 0, a) as a guiding curve.

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