20MAU14

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2022
(Fifth Semester)

Branch – **MATHEMATICS**

REAL ANALYSIS

Time: Three Hours Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 x 1 = 10)

1. If $p \in E$ and p is not a limit point of E, then p is called an __________ of E.
   (i) Closed (ii) Open
   (iii) Isolated point (iv) Interior point
2. The set of all complex z such that $|z| < 1$ is
   (i) Open, Bounded, and not closed (ii) Open perfect and bounded
   (iii) Closed perfect and bounded (iv) Open Bounded and closed
3. If F is closed and K is compact, then $F \cap K$ is ________
   (i) Compact (ii) Closed
   (iii) Open (iv) Not empty
4. Every bounded infinite subset of $\mathcal{R}^k$ has a limit point in $\mathcal{R}^k$
   (i) Weierstrass Theorem (ii) Heine Borel Theorem
   (iii) Cantor set (iv) Baire's Theorem
5. $\sum \frac{1}{n^p}$ converges if
   (i) $p < 1$ (ii) $p \leq 1$
   (iii) $p > 1$ (iv) $p \geq 1$
6. If $\sum a_n$ converges absolutely, then __________
   (i) $\sum|a_n|$ diverges (ii) $\sum|a_n|$ converges
   (iii) $\sum a_n$ converges (iv) $\sum|z_n|$ converges
7. A mapping f of a set E into $\mathcal{R}^k$ is said to be __________ if there is a real number M such that $|f(x)| \leq M$ for all $x \in E$
   (i) Bounded (ii) Closed
   (iii) Open set (iv) Co-ordinate functions
8. For the function $f(x) = \begin{cases} x, x \text{ is rational} \\ 0, x \text{ is irrational} \end{cases}$
   (i) f has the discontinuity of the second kind at $x = 0$
   (ii) f has the discontinuity of the second kind at every other point
   (iii) f is continuous at $x = 0$ and has a discontinuity of the second kind at every other point
   (iv) f is continuous at $x = 0$
9. Let f be defined on $[a, b]$ if f has a __________ at a point $x \in (a, b)$ and if $f'(x)$ exists then $f'(x) = 0$
   (i) Local maximum (ii) Local minimum
   (iii) Maximum (iv) Minimum

Cont...

10. Let f be defined on $[a, b]$. If f is differentiable at a point $x \in [a, b]$ then f is ___________ at x

(i) Differentiable (ii) not differentiable
(iii) continuous (iv) not continuous

SECTION - B (25 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** Marks (5 x 5 = 25)

11 a Prove that every neighborhood is an open set.
OR
b Prove that a set E is open if and only if its complement is closed.

12 a Prove that compact subsets of metric spaces are closed.
OR
b Let p be a non-empty perfect set in $\mathcal{R}^k$ then prove that P is uncountable.

13 a Prove that the sub sequential limit of a sequence $\{p_n\}$ is a metric space X form a closed subset of X.
OR
b State and prove Root test.

14 a Prove that a mapping f of a metric space X into a metric space Y is continuous on X if and only if $f^{-1}(v)$ is open in X for every open set V in Y.
OR
b Suppose f is a continuous mapping of a compact metric space X into a metric space Y. Then prove that $f(x)$ is compact.

15 a Show that the function f be defined by $f(x) = \begin{cases} x \sin\frac{1}{x}, x \neq 0 \\ 0, \ x = 0 \end{cases}$ is not derivable at $x = 0$.
OR
b State and prove a Generalized Mean value Theorem.

SECTION -C (40 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** Marks (5 x 8 = 40)

16 a Let $\{E_n\}$ $n = 1,2,3, \ldots\ldots$ be a sequence of countable sets and put $= \cup_{n=1}^{\infty} E_n$, then prove that S is countable.
OR
b Define neighborhood and limit point.
c) If X is a metric space and $E \subset X$ then prove that
i) $\bar{E}$ is closed
ii) $E = \bar{E}$ if and only if E is closed
iii) $\bar{E} \subset F$ for every closed set $F \subset X$ such that $E \subset F$

17 a Suppose $K \subset Y \subset X$, prove that K is compact relative to X if and only if K is compact relative to Y.
OR
b State and prove Heine-Borel Theorem.

18 a Suppose $\{S_n\}$ is monotonic, then prove that $\{S_n\}$ converges if and only if it is bounded.

b If $p > 0$ and $\propto$ is real then prove that $\lim_{n\to\infty}\frac{n^{\propto}}{(1+p)^n} = 0$

OR

c) Prove that $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$

d) Prove that e is irrational

19 a Let f be a continuous mapping of a compact metric space X in to a metric space Y, then prove that f is uniformly continuous on X.

OR

b Suppose f is a continuous mapping of a compact metric space X into a metric space Y, then prove that $f(x)$ is compact.

c If f is a continuous mapping of a metric space X into a metric space Y and if E is a connected subset of X, then prove that $f(E)$ is connected.

20 a State and prove the chain rule for differentiation.

OR

b State and prove Taylor's Theorem.

Z-Z-Z END