PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2022
(Fourth Semester)

Branch – **MATHEMATICS**

NUMBER THEORY

Time: Three Hours | Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 x 1 = 10)

1. If the successor of two elements $a, b \in \mathbb{N}$ are equal then _____.
(i) $a < b$ (ii) $a > b$ (iii) $a = b$ (iv) $a \neq b$.

2. (ma, m(ii)= _____.
(i)(a. (ii) (ii)m(a, (ii) (iii)[a, b] (iv)m[a, b].

3. If p is a prime and p+2 is also prime then they are called _____.
(i)twin primes (ii) coprimes
(iii)Siamese twins (iv) composite.

4. The number, sum of the divisors of 45 is __, ___.
(i)6, 72 (ii)6, 78 (iii)5, 72 (iv)5, 78.

5. "The integer $2^{2^n} - 1$ is a prime for all $n \geq 0$" this conjecture is due to _____.
(i)Gold bach (ii)Dirichlet (iii)Euler (iv)Fermat.

6. If $(a, m) = 1$ then the congruence $ax \equiv b \pmod{m}$ has _____ solution.
(i)only one (ii)more than one
(iii)no (iv)two

7. If $(a, n) = 1$ then $a^{\Phi(n)} \equiv 1 \pmod{n}$ this theorem is due to _____.
(i)Euler (ii)Fermat (iii)Euler-Fermat (iv)Wilson.

8. The two solutions of the congruence $x^2 \equiv 1 \pmod{p}$, where p is a prime number are ___.
(i)1, p (ii)−1, p (iii)−1, p − 1 (iv)1, p − 1.

9. Let (x, y, z) be a positive solution of $x^2 + y^2 = z^2$ then both x and y _____.
(i)odd (ii)even (iii)cannot be odd (iv)none of these.

10. For the equation $x^2 + y^2 = n$, Q(n) denotes number of______.
(i)solutions (ii)primitive solutions
(iii)non-negative primitive solutions (iv)negative primitive solutions.

SECTION - B (25 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** Marks (5 x 5 = 25)

11.a) State and prove Trichotomy law on natural numbers.

Or

b) Prove that $[a, b] = \frac{ab}{(a,\ b)}$.

12.a) Prove: If a and b are integers and p is a prime such that $p|ab$ then either $p|a$ or $p|b$.

Or

b) Find the highest power of 7 dividing 1000!.

Cont...

13.a) State and prove Gap theorem.

Or

b) If $ac \equiv bc \pmod{m}$ then $a \equiv b \left(\mathrm{mod}\ \frac{m}{d}\right)$ where $d = (c, m)$.

14.a) State and prove Fermat's theorem.

Or

b) Solve: $15x \equiv 6 \pmod{21}$.

15.a) Prove: If the positive integers x, y, z satisfy the equation $x^2 + y^2 = z^2$ then at least one of x, y, z is divisible by 5.

Or

b) Show that the function $\frac{N(n)}{4}$ is multiplicative.

SECTION -C (40 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 8 = 40)

16.a) i. Prove by Mathematical induction that $1^3 + 2^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$.(5)

ii. Prove that $(a^m)^n = a^{mn}$, $a, m, n \in \mathbb{N}$.(3)

Or

b) State and prove division algorithm.

17.a) If $a + b \neq 0$, $(a, b) = 1$ and p is an odd prime. Prove that $\left(a + b, \frac{a^p+b^p}{a+b}\right) = 1$ or p.

Or

b) State and prove Mobius inversion formula.

18.a) Prove that Fermat numbers are coprimes.

Or

b) Find the remainder when 41^{65} is divisible by 7.

19.a) State and prove Wilson's theorem.

Or

b) Find the remainder when 18! is divided by 437.

20.a) Obtain the solution of $x^2 + y^2 = z^2$.

Or

b) Find all Pythagorean triples (x, y, z) with $x < y$ and $z = 481$. Which of these triples are primitive?

Z-Z-Z END