20MAU13 / 18MAU13

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2022
(Fifth Semester)

Branch – **MATHEMATICS**

ALGEBRA

Time: Three Hours   Maximum: 75 Marks

SECTION-A (10 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** marks   (10 x 1 = 10)

1 If S is a nonempty set, then A(S) is the set of all____.
(i) identity mapping of S onto itself   (ii) 1-1 mapping of S onto itself
(iii) onto mapping of S onto itself   (iv) none of these

2 If G is a group of even order, then it has an element $a \neq e$ satisfying $a^2 =$_.
(i) a^{-1} (ii) a
(iii) e   (iv) 2

3 Every subgroup of an abelian group is______.
(i) right coset   (ii) left coset
(iii) normal   (iv) quotient group

4 If ϕ is a homomorphism of G into $\overline{G}$, then (e) =___.
(i) e (ii) $\bar{e}$
(iii) e^{-1} (iv) 1

5 If H is a subgroup of G, for every $g \in G$, gHg^{-1} is_of G.
(i) subgroup   (ii) normal subgroup
(iii) quotient subgroup   (iv) cyclic subgroup

6 $(1\ 2\ 4\ 7)^{-1}$ =_______.
(i) (2 1 4 7)   (ii) (1 2 7 4)
(iii) (2 1 4 7)   (iv) (7 4 2 1)

7 If R is the ring of integers mod 7, then R is a
(i) commutative ring   (ii) non commutative ring
(iii) integral domain   (iv) field

8 Any field is an ______
(i) integral domain   (ii) zero divisor
(iii) normal   (iv) perfect

9 Every integral domain can be imbedded in a ______.
(i) ring   (ii) commutative ring
(iii) division ring   (iv) field

10 If $a|b$ and $a|c$, then____.
(i) $a|\ (a \pm b)$ (ii) $a|(b \pm c)$
(iii) $b|c$ (iv) $b|a$

Cont...

SECTION - B (25 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 5 = 25)

11 a Given G is a group, then prove the following
(i) The identity element of G is unique (ii) For every $a \in G$ has a unique inverse in G.

OR

b If H is a non-empty finite subset of a group G and H is closed under multiplication, then prove that H is a subgroup of G.

12 a Prove that HK is a subgroup of G if and only if HK = KH.

OR

b If ϕ is a homomorphism of G into $\overline{G}$ with kernel K, then prove that K is a normal subgroup of G.

13 a Let G be a group and ϕ is an automorphism of G. If $a \in G$ is of order $(a) > 0$, then prove that $o(\phi(a)) = o(a)$.

OR

b Find the orbit and cycle of the following permutation.

(i) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 3 & 4 & 5 & 1 & 6 & 7 & 9 & 8 \end{pmatrix}$ (ii) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 5 & 4 & 3 & 1 & 2 \end{pmatrix}$

14 a Define ring with 2 examples.

OR

b Given ϕ is a hommormorphism of R into R', then prove that $\phi(0) = 0$ and $\phi(-a) = -\phi(a)$ for $a \in R$.

15 a Given R is a commutative ring with unit element, whose only ideals are (0) and Ritself. Then prove that R is a field.

OR

b Given R is a Euclidean ring. Suppose that for $a, b, c \in R$, $a|bc$ but $(a, b) = 1$, then prove that $a|c$.

SECTION -C (40 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 8 = 40)

16 a Let G be the set of all 2 x 2 matrices $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ where a, b, c, d are real numbers such that $ad - bc \neq 0$. Then show that G is a non-abelian group

OR

b State and prove Lagrange's theorem.

17a Given H and K are finite subgroups of G with orders (H) and (K) respectively, then prove that $(HK) = \frac{o(H)o(K)}{O(H \cap K)}$

OR

b State and prove Cauchy's theorem for abelian group.

18 a State and prove Cayley's theorem.

OR

b Prove that every permutation is a product of 2 – cycles.

19 a Prove that a finite integral domain is a field.

OR

b Given U is an ideal of the ring R, then prove that R / U is a ring and is a homomorphic image of R.

20 a If R is a commutative ring with unit element and M is an ideal of R, then prove thatM is a maximal ideal of R if and only if R / M is a field.

OR

b State and prove unique factorization theorem.

Z-Z-Z

END