20MAP10

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION MAY 2022
(Second Semester)

Branch – **MATHEMATICS**

MEASURE THEORY AND INTEGRATION

Time: Three Hours | Maximum: 50 Marks

SECTION-A (5 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 1 = 5)

1 _____ measurable.
(i) Union of a countable collection of measurable
(ii) complements of a measurable
(iii) Union of a countable collection of measurable
(iv) all

2 Let f be non negative measurable function on E then $\int_E f = 0$ iff_______.
(i) $f \le 0$ $a.e\ on\ E$ (ii) $f = 0$ $a.e\ on\ E$
(iii) $f \ge 0$ $a.e\ on\ E$ (iv) $f > 0$ $a.e\ on\ E$

3 The Lebesgue set of a function f ε L(a,b) contains any point at which f is_____.
(i) Discontinuous (ii) not compact
(iii) continuous (iv) not continuous

4 A is a positive set with respect to V and if for $E \in S$, $\mu(E) =$ __________ then μ is a measure
(i) $v(E)$ (ii) $v(A)$
(iii) $v(E \cap A)$ (iv) $v(A \cup E)$

5 Every outer measure induced by a measure on algebra is ____.
(i) Regular (ii) regular outer measure
(iii) σ-algebra (iv) complete measure

SECTION - B (15 Marks)

Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 3 = 15)

6 a Prove that every integral is measurable.

OR

b Prove that the class M is a σ-algebra.

7 a State and prove Lebesgue monotonic convergence theorem.

OR

b Analyze the statement of Lebesgue's Dominated convergence theorem.

8 a If f is absolutely continuous on $[a,b]$ then prove that it is of bounded variation on $[a,b]$.

OR

b Prove that let [a,b] be a finite interval and let $f \in L(a,b)$ with indefinite integral F then $F' = f$ $a.e$ in [a,b].

Cont...

9 a Analyze the statement, Let v be a signed measure on the measurable space (X,B) then there is a positive set A and a negative set B such that $X = AUB$ and $A \cap B = \phi$. The pair A,B is said to be a Hahn decomposition of X with respect to v. It is unique to the extent that A_1, B_1 and A_2, B_2 are Hahn decomposition of X with respect to v then $A_1 \Delta A_2$ is a $v-$ null set.

OR

b Analyze the statement, let v be a signed measure on [[X,S]]. Then there exists measures v+and V- on [[X,S]] such that $v = v^+ - v^-$ And $v^+ \perp v^-$. The measures v^+ and v^- are uniquely defined by v and $v = v^+ - v^-$ is said to be Jordan decomposition of v.

10 a Let f be a non-negative $S \times T$-measurable function and let $\phi(x) = \int_Y f_X dv, \psi(y) = \int_X f^y \, d\mu$ for each $x \in X, y \in Y$ then prove that ϕ is $S-$measurable, ψ is $T-$measurable and $\int_X \phi \, d\mu = \int_{X \times Y} f \, d(\mu \times v) = \int_y \psi \, dv$

OR

b Let x be a point of X and E a set in $R_{\sigma\delta}$ then prove that E_x is a measurable subset of Y.

SECTION -C (30 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 6 = 30)

11 a Prove that not every measurable set is a Borel set, each open set and each closed set is measurable.

OR

b Prove that the outer measure of an interval equals its length.

12 a State and prove Fatou's lemma.

OR

b Let f and g be integrable functions then prove the following.

(i) af is integrable and $\int af \, dx = a\int f \, dx$

(ii) $f + g$ is integrable and $\int (f+g)dx = \int fdx + \int gdx$

(iii) If $f = 0$ a.e then $\int fdx = 0$

(iv) If $f \le g$ $a.e$ then $\int f \, dx \le \int g \, dx$

If A and B are disjoint measurable sets then $\int_A f \cdot dx + \int_B f \, dx = \int_{AUB} f \, dx$

13 a Prove that let [a,b] be a finite interval and let $f \in L(a,b)$ with indefinite integral F then $F' = f$ $a.e$ in [a,b].

OR

b State and prove Vitali's theorem.

14 a Analyze the statement of Radon-Nikodym theorem.

OR

b Analyze the statement of Hahn decomposition theorem.

15 a State and prove Fubini's theorem.

OR

b State and prove extension theorem.

Z-Z-Z

END