20MAP08

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION MAY 2022
(Second Semester)

Branch - **MATHEMATICS**

TOPOLOGY

Time: Three Hours Maximum: 50 Marks

SECTION-A (5 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 1 = 5)

1. If X is any set, the collection of all one-point subsets of X is a basis for the -----
(i) Order topology (ii) Metric topology
(iii) Discrete topology (iv) Standard topology

2. X and Y are topological spaces, $f: X \rightarrow Y$ is continuous if and only if for every closed set B of Y, the set $f^{-1}(B)$ is -------- in X
(i) Open (ii) Closed
(iii) Continuous (iv) CLO open

3. Each closed interval in a simply ordered set having the least upper bound property in the order topology is______.
(i) Compact (ii) Open (iii) Sequentially Compact (iv) Closed

4. If for each pair A,B of disjoint closed sets of a space X, there exist disjoit open sets containing A and B, then X is________.
a) Regular (ii) Normal (iii) Hausdorff (iv) Lindelof

5. Every regular space with a countable basis is metrizable is given by_________.
(i) Urysohn lemma (ii) Ursohn metrization theorem
(iii)Tietze Extension Theorem (iv) Tychonoff Theorem

SECTION - B (15 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 3 = 15)

6. (a) Let X be a set and $\mathcal{B}$ be a basis for a topology τ on X. Prove that τ equals the collection of all union of elements of $\mathcal{B}$.

(OR)

(b) Show that every finite point set in a Hausdorff space is closed.

7. (a) If $f: X \rightarrow Y$ is continuous then show that for every subset A of X, $f(\overline{A}) \subset \overline{f(A)}$.

(OR)

(b)State and prove Pasting lemma.

8. (a) If the sets C and D form a separation of X, and if Y is a connected subspace of X, prove that Y lies entirely within either C or D.

(OR)

(b)Show that every closed subspace of a compact space is compact.

9. (a). State the two countability axioms.

(OR)

Cont...

(b) Let X be a topological space and let one-point sets in X be closed. If X is regular then show that given a point x of X and a neighbourhood U of x, there is a neighbourhood V of x such that $\overline{V} \subset U$.

10. (a) Show that a subspace of a completely regular space is completely regular.

(OR)

(b) Prove that every metrizable space is normal.

SECTION -C (30 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** Marks (5 x 6 = 30)

11 (a) (i) Define a topological space. (2)

(ii) Let A be a subset of the topological space X; let A' be the set of all limit points of A. Then prove that $\overline{A} = A \cup A'$ (4)

(OR)

(b) Let Y be a subspace of X. Then prove that a set A is closed in Y if and only if it equals the intersection of a closed set of X with Y.

12 (a) Let $f : A \to X \times Y$ be given by the equation

$$f(a) = (f_1(a), f_2(a)).$$

Then prove that f is continuous if and only if the functions

$f_1 : A \to X$ and $f_2 : A \to Y$

are continuous.

(OR)

(b) State and prove sequence comma.

13 (a) Prove that the cartesian product of two connected spaces is connected.

(OR)

(b) Prove that a subspace A of R^n is compact if and only if it is closed and is bounded in the euclidean metric d or the squae metric ρ.

14 (a) Suppose that X has a countable basis. Then prove that the following:

(i) Every open covering of X contains a countable subcollection covering X.

(ii) There exists a countable subset of X that is dense in X.

(OR)

b) Prove that a subspace of a regular space is regular; a product of regular spaces is regular.

15 (a) Prove that every compact Hausdoff space is normal.

(OR)

(b) Let X be a set; let D be a collection of subsets of X that is maximal with espect to the finite intersection property. Then prove the following:

(i) Any finite intersection of elements of D is an element of D.

(ii) If A is a subset of X that intesects every element of D, then A is an element of D.

Z-Z-Z END