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PSG COLLEGE OF ARTS & SCIENCE

(AUTONOMOUS)

MSc DEGREE EXAMINATION MAY 2022

(First Semester)

Branch – **MATHEMATICS**

REAL ANALYSIS

Time: Three Hours — Maximum: 50 Marks

SECTION-A (5 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** marks (5 x 1 = 5)

1. If $f_1(x) \le f_2(x)$ on [a,b], then $\int_a^b f_1 d\alpha$ ______ $\int_a^b f_2 d\alpha$

   a) b) $\le$ c) $\ge$ d) $\neq$

2. If $\{f_n\}$ is sequence of continuous functions on E, and if $f_n \to f$ uniformly on E, then f is _______ on E.

   a) connected b)continuous c)compact d)unbounded

3. $\Gamma(8)$ = _________

   a)56 b)7! c)8! d) 9!

4. Let r a positive integer. If a vector space X is spanned by a set of r vectors, then dim X ____ r.

   a) $\le$ b)= c) $\ge$ d) $\neq$

5. If $\mu(E) = 0$ and f is measurable then $\int_E f \, d\mu =$ ______.

   a)1 b)-1 c)2 d) 0

SECTION - B (15 Marks)

Answer **ALL** Questions

ALL Questions Carry **EQUAL** Marks (5 x 3 = 15)

6. a) If f is continuous on $[a,b]$ then prove that $f \in \Re(\alpha)$ on $[a,b]$

   (or)

   b) If $f \in \Re(\alpha)$ and $g \in \Re(\alpha)$ on $[a,b]$, then prove that $(i)\ fg \in \Re(\alpha)$ $(ii)\ |f| \in \Re(\alpha)$ and $\left|\int_a^b f\, d\alpha\right| \le \int_a^b |f|\, d\alpha.$

7. a) State and prove the Cauchy criterion for uniform convergence.

   (or)

   b) If K is a compact metric space, if $f_n \in$ □ (K) for n=1,2,3,..., and if $\{f_n\}$ converges uniformly on K, then prove that $\{f_n\}$ is equi-continuous on K.

Cont...

8. a) Given a double sequence $\{a_{ij}\}$, suppose that $\sum_{j=1}^{\infty}|a_{ij}| = b_i$ and $\sum b_i$ converges. Then prove that $\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} a_{ij} = \sum_{j=1}^{\infty}\sum_{i=1}^{\infty} a_{ij}$.

(or)

b) If f is a positive function on $(0,\infty)$ such that

$(i)\ f(x+1) = xf(x)$

$(ii)\ f(1) = 1$

$(iii)\ \log f$ is convex, then prove that $f(x) = \Gamma(x)$.

9. a) Prove that a linear operator A on a finite-dimensional vector space X is one-to-one if and only if the range of A is all of X.

(or)

b) Suppose f maps a convex open set $E \subset R^n$ into R^m, f is differentiable in E, and there is a real number M such that $\|f'(x)\| \le M$ for every $x \in E$. Then prove that $|f(b) - f(a)| \le M|b-a|$ for all $a \in E, b \in E$.

10. a) If f and g are measurable real-valued functions defined on X, if F is real and continuous on R^2, and put $h(x) = F(f(x), g(x))$. Then prove that h is measurable.

(or)

b) State and prove Lebesgue's dominated convergence theorem.

SECTION -C (30 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 6 = 30)

11. a) Assume α increases monotonically and $\alpha' \in \Re$ on [a,b]. Let f be a bounded real function on [a,b]. Then if and only if $f\alpha' \in \Re$. In that case $\int_a^b f d\alpha = \int_a^b f(x)\alpha' dx$.

(or)

b) If $f \in \Re$ on [a,b]. For $a \le x \le b$, put $F(x) = \int_a^x f(t)dt$. Then F is continuous on [a,b]; if f is continuous at a point x_0 of [a,b] then prove that F is differentiable at x_0, and $F'(x_0)=f(x_0)$.

12. a) Suppose $f_n \to f$ uniformly on a set E in a metric space. Let x be a limit point of E, suppose that $\lim_{t\to x} f_n(t) = A_n$. Then prove that $\{A_n\}$ converges and $\lim_{t\to x} f_n(t) = \lim_{n\to\infty} A_n$.

(or)

b) If K is a compact, if $f_n \in$ □ (K) for n=1,2,3,..., and if $\{f_n\}$ is pointwise bounded and equi-continuous on K, then prove that

(i) $\{f_n\}$ is uniformly bounded on K

(ii) $\{f_n\}$ contains a uniformly convergent subsequence.

13. a) Suppose $a_0,\ldots,a_n$ are complex numbers, $n\geq 1,\ a\neq 0, P(z)=\sum_0^n a_k z^k$. Then prove that P(z) = 0 for some complex number z.

(or)

b) State and prove Parseval's theorem.

14. a) Let Ω be the set of all invertible linear operators on R^n .Prove that

i) If $A\in\Omega, B\in L(R^n), and\ \|B-A\|\|A^{-1}\|<1, then\ B\in\Omega$.

ii) Ω is an open subset of L(R^n) and the mapping $A\to A^{-1}$ is continuous on Ω.

(or)

b) State and prove inverse function theorem.

15. a) Prove that $\mathfrak{M}(\mu)$ is a σ-ring and is countably additive on $\mathfrak{M}(\mu)$.

(or)

b) State and prove Lebesgue's monotone convergence theorem.

Z-Z-Z END