20MAP06

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION MAY 2022
(Second Semester)

Branch – **MATHEMATICS**

COMPLEX ANALYSIS

Time: Three Hours | Maximum: 50 Marks

SECTION-A (5 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 1 = 5)

1. The cross ratio preserves under the …….. transformation
   a) linear b) non-linear
   c) bi-linear d) none
2. In a series with analytic terms $f(z) = f_1(z) + f_2(z) + ... + f_n(z)$ converges uniformly on every ………… subset of Ω .
   a) closed b) open
   c) compact d) none
3. $\pi_{n=2}^{\infty}\left(1-\frac{1}{n^2}\right)$ converges to ………………….
   a) 1 b) $\frac{1}{2}$
   c) $\frac{1}{3}$ d) $\frac{1}{4}$
4. An analytic function g(z) is said to be univalent if ………………
   a) $g(z_1) > g(z_2)$ b) $g(z_1) < g(z_2)$
   c) $g(z_1) = g(z_2)$ d) none
5. $\sigma(z)$ is not an ……………. function.
   a) analytic b) elliptic
   c) entire d) none.

SECTION - B (15 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 3 = 15)

6 a Prove that a linear transformation carries circles into circles.

OR

b Prove that $n(\gamma, a) = n(\gamma, b)$ and $n(\gamma, a) = 0$ if 'a' lies outside γ .

7 a State and prove Cauchy's integral formula.

OR

b State and prove residue theorem.

8 a State and prove Hurtwitz theorem.

OR

b Prove that necessary and sufficient condition for $\sum|\log(1+a_n)|$ to be convergent is that $\sum|a_n|$ is convergent.

9 a Let f be a topological mapping of the region Ω onto the region Ω'. If the sequence $\{z_n(t)\}$ or {z(t)} tend to the boundary of Ω. Prove that $\{f_n(z_n)\}$ or $\{f(z(t)\}$ tend to the boundary of Ω'.

OR

b State and prove Harnack's inequality.

Cont...

10 a Prove that the sum of residues of an elliptic function and its poles inside any cell is zero.

OR

b Prove that $\wp(z)$ is an elliptic function.

SECTION -C (30 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 6 = 30)

11 a The line integral $\int_{\gamma} pdx + qdy$ defined in Ω depends only on end points on Ω. Prove that a function U(x,y) in Ω with a partial derivatives $p = \frac{\partial u}{\partial x}, q = \frac{\partial u}{\partial y}$.

OR

b Suppose that $\phi(\zeta)$ is continuous on arc γ. Prove that the function $F_n(z) = \int_{\gamma} \frac{\phi(\zeta)}{(\zeta - 2)^n} d\zeta$ is analytic in each of the region determined by γ and its derivative $F_n'(z) = nF_{n+1}(z)$.

12 a State and prove Residue theorem.

OR

b Suppose that f (z) is analytic for $\gamma_1 < |z| < \gamma_2$ and set $M(r) = \max\{|f(z)| : |z| = r\}$. Prove that $M(r) \leq [M(r_1)]^{\alpha} [M(r_2)]^{1-\alpha}$.

13 a State and prove Mittag-Leffler's theorem.

OR

b State and prove Jensen's formula.

14 a State and prove Riemann Mapping theorem.

OR

b State and prove Schwartz-Christoffel's formula.

15 a Prove that a discrete module consists either of zero alone or an integral multiples of nW, of a single complex number W ≠ 0 are of linear combinations of two numbers W_1 and W_2 with non- real ratio $\frac{W_2}{W_1}$.

OR

b Prove that the sum of zero's – the sum of poles are an elliptic function inside any cell is a period.

Z-Z-Z END