(AUTONOMOUS)

MSc DEGREE EXAMINATION MAY 2026
(Second Semester)

Branch – **STATISTICS**

ESTIMATION THEORY

Time: Three Hours | Maximum: 75 Marks

SECTION-A (10 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	An estimator whose expected value equals the true parameter is called ______ (i) consistent estimator (ii) efficient estimator (iii) unbiased estimator (iv) sufficient estimator	K1	CO1
	2	The Factorization Theorem is used to identify______________ (i) completeness (ii) sufficiency (iii) efficiency (iv) consistency	K2	CO1
2	3	Recall the outcome of the Cramer–Rao inequality. (i) upper bound for variance (ii) lower bound for variance (iii) mean squared error (iv) confidence interval	K1	CO2
	4	The Rao–Blackwell theorem helps in obtaining _________ (i) biased (ii) minimum variance unbiased estimator (iii) maximum likelihood estimator (iv) consistent estimator	K2	CO2
3	5	In the Method of Moments, parameters are estimated by equating ____ (i) variances (ii) sample moments with population moments (iii) likelihood functions (iv) derivatives	K1	CO3
	6	The estimator obtained by maximizing the likelihood function is called ______ (i) Bayes estimator (ii) Method of Moments estimator (iii) Maximum Likelihood estimator (iv) Efficient estimator	K2	CO3
4	7	A family of distributions written in exponential form is called _____ (i) Normal family (ii) Scale family (iii) Exponential family (iv) Location family	K1	CO4
	8	A Bayes estimator is obtained by minimizing __________ (i) likelihood (ii) posterior risk (iii) variance (iv) bias	K2	CO4
5	9	A confidence interval constructed using small sample theory for variance follows _________ (i) Normal distribution (ii) t-distribution (iii) Chi-square distribution (iv) F-distribution	K1	CO5
	10	The degrees of freedom for confidence interval of ratio of variances are _________ (i) $n_1 + n_2 - 2$ (ii) $n_1 - 1, n_2 - 1$ (iii) n_1, n_2 (iv) $n_1 - n_2$	K2	CO5

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SECTION - B (35 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	Explain the concepts of unbiasedness and consistency of an estimator. Illustrate with suitable examples.	K3	CO1
		(OR)		
	11.b.	Let $X_1, X_2, \ldots, X_n$ be a random sample from a distribution with mean μ and variance σ^2. (a) Show that the sample mean $\bar{X}$ is an unbiased estimator of μ. (b) Prove that $\bar{X}$ is consistent.		
2	12.a.	State the Cramér–Rao inequality and Explain the conditions under which equality holds.	K3	CO2
		(OR)		
	12.b.	Let $X_1, \ldots, X_n \sim N(\mu, \sigma^2)$, where σ^2 is known. (a) Find the Fisher information for μ. (b) Obtain the Cramér–Rao lower bound. (c) Verify whether $\bar{X}$ is efficient.		
3	13.a.	Let $X_1, X_2, \ldots, X_n$ be a random sample from an exponential distribution with pdf: $f(x;\theta) = \frac{1}{\theta} e^{-x/\theta}, x > 0$ (a) Obtain the Maximum Likelihood Estimator of θ. (b) Show whether the estimator is unbiased or biased.	K3	CO3
		(OR)		
	13.b.	Compare Minimum chi – square and Modified Minimum chi – square method.		
4	14.a.	Define an Exponential Family of distributions. Explain how sufficiency arises naturally.	K3	CO4
		(OR)		
	14.b.	Let $X \sim \text{Binomial}(n, p)$ and assume prior $p \sim \text{Beta}(\alpha, \beta)$. (a) Obtain the posterior distribution of p. (b) Find the Bayes estimator of p under squared error loss.		
5	15.a.	Explain the concept of confidence interval and confidence bounds. What is a uniformly most accurate confidence bound?	K3	CO5
		(OR)		
	15.b.	Let $X_1, X_2, \ldots, X_n \sim N(\mu, \sigma^2)$, where σ^2 is known. (a) Construct a $100(1-\alpha)\%$ confidence interval for μ. (b) Obtain the length of the interval.		

SECTION -C (30 Marks)

Answer **ANY THREE** questions

ALL questions carry **EQUAL** Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	Let $X_1, X_2, \ldots, X_n$ be a random sample from exponential distribution: $f(x;\theta) = \theta e^{-\theta x}, x > 0$ (a) Obtain a sufficient statistic for θ. (b) Show that it is complete. (c) Hence derive the UMVUE of θ.	K4	CO1
2	17	State and Prove the Rao–Blackwell Theorem. Explain its practical significance.	K5	CO2
3	18	Let $X_1, \ldots, X_n \sim N(\mu, \sigma^2)$, where both parameters are unknown. Obtain the MLEs of μ and σ^2.	K4	CO3
4	19	Let $X_1, \ldots, X_n \sim \text{Binomial}(n, p)$ with Beta prior. (a) Obtain the posterior distribution. (b) Derive the Bayes estimator under squared error loss. (c) Compare with MLE.	K5	CO4
5	20	Obtain $100(1-\alpha)\%$ confidence limits (large samples) for the parameters 'λ' of the Poisson distribution.	K4	CO5

Z-Z-Z END