

**OPTIMIZATION TECHNIQUES**

Time: Three Hours

Maximum: 75 Marks

**SECTION-A (10 Marks)**

Answer ALL questions

ALL questions carry EQUAL marks

(10 × 1 = 10)

| Module No. | Question No. | Question                                                                                                                                                                                                                                                                     | K Level | CO  |
|------------|--------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------|-----|
| 1          | 1            | A constraint in an LPP is expressed as<br>(i) an equation with = sign      (ii) inequality with $\geq$ sign<br>(iii) inequality with $\leq$ sign      (iv) any of the above                                                                                                  | K1      | CO1 |
|            | 2            | In Graphical method, If all the solution points of the required region is not able to show case in the graph, then the solution is said to be _____ under maximum case<br>(i) no solution      (ii) infinite solution<br>(iii) bounded solution      (iv) unbounded solution | K2      | CO1 |
| 2          | 3            | Recall "In a transportation problem, if there are more supply points than demand points, the problem is termed as<br>(i) Balanced      (ii) Unbalanced<br>(iii) Feasible      (iv) Optimal                                                                                   | K1      | CO2 |
|            | 4            | Identify the another name of Least Cost Method.<br>(i) North West Corner      (ii) Matrix Minima<br>(iii) Row Minima      (iv) Column Minima                                                                                                                                 | K2      | CO2 |
| 3          | 5            | For a two person zero-sum game, the value of game can be<br>(i) Determined only if the pay-off matrix has a saddle point<br>(ii) Positive, negative or zero<br>(iii) Determined only if the game is fair<br>(iv) None of the above                                           | K1      | CO3 |
|            | 6            | When maximin and minimax values of the game are same, then<br>(i) There is a saddle point      (ii) Solution does not exist<br>(iii) Strategies are mixed      (iv) None of the above                                                                                        | K2      | CO3 |
| 4          | 7            | Network problems have advantage in terms of project ____<br>(i) scheduling      (ii) planning<br>(iii) controlling      (iv) all the above                                                                                                                                   | K1      | CO4 |
|            | 8            | When maximin and minimax values of the game are same, then<br>(i) There is a saddle point      (ii) Solution does not exist<br>(iii) Strategies are mixed      (iv) None of the above                                                                                        | K2      | CO4 |
| 5          | 9            | Which one of the following methods is followed in critical path method?<br>(i) Deterministic      (ii) Fuzzy<br>(iii) Probabilistic      (iv) Deterministic                                                                                                                  | K1      | CO5 |
|            | 10           | In PERT, the time estimates for tasks are typically represented as:<br>(i) A single deterministic value<br>(ii) Three estimates: optimistic, pessimistic and most likely<br>(iii) A range of possible values<br>(iv) Only the average time                                   | K2      | CO5 |

**SECTION - B (35 Marks)**

Answer ALL questions

ALL questions carry EQUAL Marks

(5 × 7 = 35)

| Module No. | Question No. | Question                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | K Level          | CO              |                  |   |            |           |   |           |           |    |     |
|------------|--------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------|-----------------|------------------|---|------------|-----------|---|-----------|-----------|----|-----|
| 1          | 11.a.        | <p>Consider the following problem faced by a production planner in a soft drink plant. He has two bottling machines A and B. A is designed for 8-ounce bottles and B for 16-ounce bottles. However, each can be used on both types with some loss of efficiency. The following data is available:</p> <table><tr><th>Machine</th><th>8-ounce bottles</th><th>16-ounce bottles</th></tr><tr><td>A</td><td>100/minute</td><td>40/minute</td></tr><tr><td>B</td><td>60/minute</td><td>75/minute</td></tr></table> <p>Each machine can be run 8 hours per day, 5 days per week. Profit on an 8-ounce bottle is 25 paise and on a 16-ounce bottle is 35 paise. Weekly production of the drink cannot exceed 3,00,000 ounces and the market can absorb 25,000 8-ounce bottles and 7,000 16-ounce bottles per week. The planner wishes to maximize his profit subject, of course, to all the production and marketing restrictions.<br/><i>Formulate this as a linear programming problem.</i></p> | Machine          | 8-ounce bottles | 16-ounce bottles | A | 100/minute | 40/minute | B | 60/minute | 75/minute | K2 | CO1 |
|            | Machine      | 8-ounce bottles                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | 16-ounce bottles |                 |                  |   |            |           |   |           |           |    |     |
|            | A            | 100/minute                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | 40/minute        |                 |                  |   |            |           |   |           |           |    |     |
| B          | 60/minute    | 75/minute                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |                  |                 |                  |   |            |           |   |           |           |    |     |
| (OR)       |              |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |                  |                 |                  |   |            |           |   |           |           |    |     |
|            | 11.b.        | <p>Use the graphical method to solve the following LPP: <i>Maximize</i> <math>Z = 2x_1 + 3x_2</math>; subject to the constraints: <math>x_1 + x_2 \leq 30</math>; <math>x_1 - x_2 \geq 0</math>; <math>x_2 &gt; 3</math>; <math>0 \leq x_1 \leq 20</math> and <math>0 \leq x_2 \leq 12</math></p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                  |                 |                  |   |            |           |   |           |           |    |     |

Cont...

|       |                                                                                                                                                                                                                                                                                                                                                      |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |       |       |       |       |    |    |     |    |     |      |   |   |   |   |   |   |   |   |   |      |    |   |    |    |    |
|-------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|-------|-------|-------|----|----|-----|----|-----|------|---|---|---|---|---|---|---|---|---|------|----|---|----|----|----|
| 2     | 12.a.                                                                                                                                                                                                                                                                                                                                                |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | $D_1$ | $D_2$ | $D_3$ | $D_4$ |    | K2 | CO2 |    |     |      |   |   |   |   |   |   |   |   |   |      |    |   |    |    |    |
|       |                                                                                                                                                                                                                                                                                                                                                      | $S_1$                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | 5     | 3     | 6     | 2     | 19 |    |     |    |     |      |   |   |   |   |   |   |   |   |   |      |    |   |    |    |    |
|       |                                                                                                                                                                                                                                                                                                                                                      | $S_2$                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | 4     | 7     | 9     | 1     | 37 |    |     |    |     |      |   |   |   |   |   |   |   |   |   |      |    |   |    |    |    |
|       |                                                                                                                                                                                                                                                                                                                                                      | $S_3$                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | 3     | 4     | 7     | 5     | 34 |    |     |    |     |      |   |   |   |   |   |   |   |   |   |      |    |   |    |    |    |
|       |                                                                                                                                                                                                                                                                                                                                                      | Demand                                                                                                                                                                                                                                                                                                                                                                                                                                                                            | 16    | 18    | 31    | 25    |    |    |     |    |     |      |   |   |   |   |   |   |   |   |   |      |    |   |    |    |    |
| (OR)  |                                                                                                                                                                                                                                                                                                                                                      |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |       |       |       |       |    |    |     |    |     |      |   |   |   |   |   |   |   |   |   |      |    |   |    |    |    |
| 12.b. | A departmental head has 4 subordinates and 4 tasks to be performed. The subordinates differ in efficiency and the tasks differ in their intrinsic difficulty. His estimate of the time each man would take to perform each task is given in the matrix below. How should the tasks be allocated, one to a man so as to minimize the total man-hours? |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |       |       |       |       |    |    |     |    |     |      |   |   |   |   |   |   |   |   |   |      |    |   |    |    |    |
|       |                                                                                                                                                                                                                                                                                                                                                      | 1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | 2     | 3     | 4     |       |    |    |     |    |     |      |   |   |   |   |   |   |   |   |   |      |    |   |    |    |    |
|       | A                                                                                                                                                                                                                                                                                                                                                    | 18                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | 26    | 17    | 11    |       |    |    |     |    |     |      |   |   |   |   |   |   |   |   |   |      |    |   |    |    |    |
|       | B                                                                                                                                                                                                                                                                                                                                                    | 13                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | 28    | 14    | 26    |       |    |    |     |    |     |      |   |   |   |   |   |   |   |   |   |      |    |   |    |    |    |
|       | C                                                                                                                                                                                                                                                                                                                                                    | 38                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | 19    | 18    | 15    |       |    |    |     |    |     |      |   |   |   |   |   |   |   |   |   |      |    |   |    |    |    |
| D     | 19                                                                                                                                                                                                                                                                                                                                                   | 26                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | 24    | 10    |       |       |    |    |     |    |     |      |   |   |   |   |   |   |   |   |   |      |    |   |    |    |    |
| 3     | 13.a.                                                                                                                                                                                                                                                                                                                                                | Determine the range of value of p and q that will make the payoff element $a_{22}$ , a saddle point for the game whose payoff matrix $(a_{ij})$ is given below:                                                                                                                                                                                                                                                                                                                   |       |       |       |       |    |    |     | K2 | CO3 |      |   |   |   |   |   |   |   |   |   |      |    |   |    |    |    |
|       |                                                                                                                                                                                                                                                                                                                                                      | $\begin{matrix} & \text{Player B} \\ \text{Player A} & \begin{bmatrix} 2 & 4 & 7 \\ 10 & 7 & q \\ 4 & p & 8 \end{bmatrix} \end{matrix}$                                                                                                                                                                                                                                                                                                                                           |       |       |       |       |    |    |     |    |     |      |   |   |   |   |   |   |   |   |   |      |    |   |    |    |    |
|       |                                                                                                                                                                                                                                                                                                                                                      | (OR)                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |       |       |       |       |    |    |     |    |     |      |   |   |   |   |   |   |   |   |   |      |    |   |    |    |    |
|       |                                                                                                                                                                                                                                                                                                                                                      | Obtain the optimal strategies for both-persons and the value of the game for zero-sum two-person game whose payoff matrix is as follows:                                                                                                                                                                                                                                                                                                                                          |       |       |       |       |    |    |     |    |     |      |   |   |   |   |   |   |   |   |   |      |    |   |    |    |    |
|       |                                                                                                                                                                                                                                                                                                                                                      | $\begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 6 \\ 4 & 1 \\ 2 & 2 \\ -5 & 0 \end{bmatrix}$                                                                                                                                                                                                                                                                                                                                                                                             |       |       |       |       |    |    |     |    |     |      |   |   |   |   |   |   |   |   |   |      |    |   |    |    |    |
| 4     | 14.a.                                                                                                                                                                                                                                                                                                                                                | In a railway marshalling yard, goods trains arrive at a rate of 30 trains per day. Assuming that the inter-arrival time follows an exponential distribution and the service time distribution is also exponential with an average 36 minutes. Calculate the following:<br>i) The mean queue size (line length), and<br>ii) The probability that the queue size exceeds 10.<br>If the input of trains increases to an average 33 per day, what will be the change in (i) and (ii)? |       |       |       |       |    |    |     | K2 | CO4 |      |   |   |   |   |   |   |   |   |   |      |    |   |    |    |    |
|       |                                                                                                                                                                                                                                                                                                                                                      | (OR)                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |       |       |       |       |    |    |     |    |     |      |   |   |   |   |   |   |   |   |   |      |    |   |    |    |    |
|       |                                                                                                                                                                                                                                                                                                                                                      | Assume that the goods trains are coming in a yard at the rate of 30 trains per day and suppose that the inter-arrival times follow exponential distribution. The service time for each train is assumed to be exponential with an average of 36 minutes. If the yard can admit 9 trains at a time (there being 10 lines, one of which is reserved for shunting purpose), calculate the probability that the yard is empty and find the average queue length.                      |       |       |       |       |    |    |     |    |     |      |   |   |   |   |   |   |   |   |   |      |    |   |    |    |    |
|       |                                                                                                                                                                                                                                                                                                                                                      | 14.b.                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |       |       |       |       |    |    |     |    |     |      |   |   |   |   |   |   |   |   |   |      |    |   |    |    |    |
|       |                                                                                                                                                                                                                                                                                                                                                      |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |       |       |       |       |    |    |     |    |     |      |   |   |   |   |   |   |   |   |   |      |    |   |    |    |    |
| 5     | 15.a.                                                                                                                                                                                                                                                                                                                                                | Write down the rules for constructing the network.                                                                                                                                                                                                                                                                                                                                                                                                                                |       |       |       |       |    |    |     | K2 | CO5 |      |   |   |   |   |   |   |   |   |   |      |    |   |    |    |    |
|       |                                                                                                                                                                                                                                                                                                                                                      | (OR)                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |       |       |       |       |    |    |     |    |     |      |   |   |   |   |   |   |   |   |   |      |    |   |    |    |    |
|       |                                                                                                                                                                                                                                                                                                                                                      | A project consists of a series of tasks labelled A, B, C, D, E, F, G, H, I. The network diagram having the following constraints: $A < D, E$ ; $B, D < F$ ; $C < G$ ; $B, G < H$ ; $F, G < I$ . Find the minimum time of completion of the project and the float value                                                                                                                                                                                                            |       |       |       |       |    |    |     |    |     |      |   |   |   |   |   |   |   |   |   |      |    |   |    |    |    |
|       |                                                                                                                                                                                                                                                                                                                                                      | 15.b.                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |       |       |       |       |    |    |     |    |     |      |   |   |   |   |   |   |   |   |   |      |    |   |    |    |    |
|       |                                                                                                                                                                                                                                                                                                                                                      | <table><tr><td>Task</td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td><td>F</td><td>G</td><td>H</td><td>I</td></tr><tr><td>Time</td><td>23</td><td>8</td><td>20</td><td>16</td><td>24</td><td>18</td><td>19</td><td>4</td><td>10</td></tr></table>                                                                                                                                                                                                                            |       |       |       |       |    |    |     |    |     | Task | A | B | C | D | E | F | G | H | I | Time | 23 | 8 | 20 | 16 | 24 |
| Task  | A                                                                                                                                                                                                                                                                                                                                                    | B                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | C     | D     | E     | F     | G  | H  | I   |    |     |      |   |   |   |   |   |   |   |   |   |      |    |   |    |    |    |
| Time  | 23                                                                                                                                                                                                                                                                                                                                                   | 8                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | 20    | 16    | 24    | 18    | 19 | 4  | 10  |    |     |      |   |   |   |   |   |   |   |   |   |      |    |   |    |    |    |

### SECTION -C (30 Marks)

Answer ANY THREE questions

ALL questions carry EQUAL Marks  $(3 \times 10 = 30)$

| Module No. | Question No. | Question                                                                                                                                                                                                | K Level | CO  |
|------------|--------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------|-----|
| 1          | 16           | Use Simplex method to solve the following LPP:<br>$Max Z = 4x_1 + 10x_2$<br>Subject to the constraints<br>$2x_1 + x_2 \leq 50$ ; $2x_1 + 5x_2 \leq 100$ ; $2x_1 + 3x_2 \leq 90$ and $x_1, x_2 \geq 0$ . | K3      | CO1 |

Cont...

|        |    |                                                         |                |                |                |                |        |    |     |
|--------|----|---------------------------------------------------------|----------------|----------------|----------------|----------------|--------|----|-----|
| 2      | 17 | Approximation Method. Also obtain the optimum solution: |                |                |                |                |        | K3 | CO2 |
|        |    |                                                         | D <sub>1</sub> | D <sub>2</sub> | D <sub>3</sub> | D <sub>4</sub> | Supply |    |     |
|        |    | S <sub>1</sub>                                          | 3              | 7              | 6              | 4              | 5      |    |     |
|        |    | S <sub>2</sub>                                          | 2              | 4              | 3              | 2              | 2      |    |     |
|        |    | S <sub>3</sub>                                          | 4              | 3              | 8              | 5              | 3      |    |     |
| Demand | 3  | 3                                                       | 2              | 2              |                |                |        |    |     |

|   |    |                                                                                                  |          |  |  |  |  |    |     |
|---|----|--------------------------------------------------------------------------------------------------|----------|--|--|--|--|----|-----|
| 3 | 18 | Solve the following game:                                                                        |          |  |  |  |  | K3 | CO3 |
|   |    | Player A                                                                                         | Player B |  |  |  |  |    |     |
|   |    | $\begin{pmatrix} 3 & 2 & 4 & 0 \\ 3 & 4 & 2 & 4 \\ 4 & 2 & 4 & 0 \\ 0 & 4 & 0 & 8 \end{pmatrix}$ |          |  |  |  |  |    |     |

|   |    |                                                                                                                                                                                                                |  |  |  |  |  |    |     |
|---|----|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|--|--|--|--|----|-----|
| 4 | 19 | A supermarket has two persons serving at the counters. The customers arrive in a Poisson fashion at the rate of 12 per hour. The service time for each customer is exponential with a mean of 6 minutes. Find: |  |  |  |  |  | K3 | CO4 |
|   |    | i) The probability that an arriving customer has to wait for service.<br>ii) The average number of customers in the system.<br>iii) The average time spent by a customer in the supermarket.                   |  |  |  |  |  |    |     |

|                                                                                                                                                                                                   |        |                                                                                                                                 |                 |                  |        |                  |      |      |      |      |      |   |        |        |        |        |        |  |  |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------|---------------------------------------------------------------------------------------------------------------------------------|-----------------|------------------|--------|------------------|------|------|------|------|------|---|--------|--------|--------|--------|--------|--|--|
| 5                                                                                                                                                                                                 | 20     | A small project is composed of seven activities whose time estimates are listed in the table as follows:                        |                 |                  |        |                  |      | K3   | CO5  |      |      |   |        |        |        |        |        |  |  |
|                                                                                                                                                                                                   |        | Activity (i-j)                                                                                                                  | Optimistic (to) | Most likely (tm) |        | Pessimistic (tp) |      |      |      |      |      |   |        |        |        |        |        |  |  |
|                                                                                                                                                                                                   |        | 1-2                                                                                                                             | 1               | 1                |        | 7                |      |      |      |      |      |   |        |        |        |        |        |  |  |
|                                                                                                                                                                                                   |        | 1-3                                                                                                                             | 1               | 4                |        | 7                |      |      |      |      |      |   |        |        |        |        |        |  |  |
|                                                                                                                                                                                                   |        | 1-4                                                                                                                             | 2               | 2                |        | 8                |      |      |      |      |      |   |        |        |        |        |        |  |  |
|                                                                                                                                                                                                   |        | 2-5                                                                                                                             | 1               | 1                |        | 1                |      |      |      |      |      |   |        |        |        |        |        |  |  |
|                                                                                                                                                                                                   |        | 3-5                                                                                                                             | 2               | 5                |        | 14               |      |      |      |      |      |   |        |        |        |        |        |  |  |
|                                                                                                                                                                                                   |        | 4-6                                                                                                                             | 2               | 5                |        | 8                |      |      |      |      |      |   |        |        |        |        |        |  |  |
|                                                                                                                                                                                                   |        | 5-6                                                                                                                             | 3               | 6                |        | 15               |      |      |      |      |      |   |        |        |        |        |        |  |  |
|                                                                                                                                                                                                   |        | a) Draw the project network.                                                                                                    |                 |                  |        |                  |      |      |      |      |      |   |        |        |        |        |        |  |  |
|                                                                                                                                                                                                   |        | b) Find the expected duration and variance of each activity. What is the expected project length?                               |                 |                  |        |                  |      |      |      |      |      |   |        |        |        |        |        |  |  |
|                                                                                                                                                                                                   |        | c) Calculate the variance and standard deviation of project length. What is the probability that the project will be completed: |                 |                  |        |                  |      |      |      |      |      |   |        |        |        |        |        |  |  |
| i. At least 4 weeks earlier than expected?                                                                                                                                                        |        |                                                                                                                                 |                 |                  |        |                  |      |      |      |      |      |   |        |        |        |        |        |  |  |
| ii. No more than 4 weeks later than expected?                                                                                                                                                     |        |                                                                                                                                 |                 |                  |        |                  |      |      |      |      |      |   |        |        |        |        |        |  |  |
| d) If the project due date is 19 weeks, what is the probability of meeting the due date?                                                                                                          |        |                                                                                                                                 |                 |                  |        |                  |      |      |      |      |      |   |        |        |        |        |        |  |  |
| Given Probability Table                                                                                                                                                                           |        |                                                                                                                                 |                 |                  |        |                  |      |      |      |      |      |   |        |        |        |        |        |  |  |
| <table><tr><td>z</td><td>0.50</td><td>0.67</td><td>1.00</td><td>1.33</td><td>2.00</td></tr><tr><td>p</td><td>0.3085</td><td>0.2514</td><td>0.1587</td><td>0.0918</td><td>0.0228</td></tr></table> |        |                                                                                                                                 |                 |                  |        | z                | 0.50 | 0.67 | 1.00 | 1.33 | 2.00 | p | 0.3085 | 0.2514 | 0.1587 | 0.0918 | 0.0228 |  |  |
| z                                                                                                                                                                                                 | 0.50   | 0.67                                                                                                                            | 1.00            | 1.33             | 2.00   |                  |      |      |      |      |      |   |        |        |        |        |        |  |  |
| p                                                                                                                                                                                                 | 0.3085 | 0.2514                                                                                                                          | 0.1587          | 0.0918           | 0.0228 |                  |      |      |      |      |      |   |        |        |        |        |        |  |  |

Z-Z-Z                      END

