

MAJOR ELECTIVE COURSE – I COMPUTATIONAL METHODS (THEORY)

Time: Three Hours

Maximum: 75 Marks

SECTION-A (10 Marks)

Answer ALL questions

ALL questions carry EQUAL marks (10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	In Gauss–Jacobi method, the values used for computing the next iteration are _____ a. Newly computed values immediately b. Only previous iteration values c. Average of old and new values d. Random values	K1	CO1
	2	The power method is used to find _____ a. Smallest eigenvalue b. All eigenvalues c. Dominant eigenvalue d. Determinant	K2	
2	3	The Trapezium rule is based on approximating the curve by _____ a. Parabola b. Cubic polynomial c. Circle d. Straight line	K1	CO2
	4	Gauss–Legendre integration is an example of _____ a. Newton–Cotes formula b. Closed formula c. Gaussian quadrature d. Finite difference method	K2	
3	5	An initial value problem (IVP) requires _____ a. Only boundary conditions b. Only final condition c. Initial condition at one point d. No condition	K1	CO3
	6	The order of the classical Runge - Kutta method is _____ a. 1 b. 2 c. 3 d. 4	K2	
4	7	If $B^2 - 4AC = 0$, the PDE is _____ a. Parabolic b. Linear c. Elliptic d. Hyperbolic	K1	CO4
	8	Laplace equation is given by _____ a. $u_{xx} = u_{yy}$ b. $u_{xx} = u_{tt}$ c. $u_{xx} - u_{yy} = 0$ d. $u_{xx} + u_{yy} = 0$	K2	
5	9	In the finite difference method, time derivative in the heat equation is usually approximated by _____ difference. a. Forward b. Backward c. Central d. Exact	K1	CO5
	10	The wave equation is _____ a. $u_t = c^2 u_{xx}$ b. $u_t = c u_{xx}$ c. $u_{tt} = c^2 u_{xx}$ d. $u_{tt} = c u_{xx}$	K2	

SECTION - B (35 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11a)	Solve the system of equations $4x_1 + x_2 + x_3 = 2$, $x_1 + 5x_2 + 2x_3 = -6$ and $x_1 + 2x_2 + 3x_3 = -4$ using Jacobi Iteration Method. Take the Initial approximation as $x^{(0)} = [0.5, -0.5, -0.5]^T$ and perform three iterations.	K2	CO1
		(OR)		
	11 b)	Consider the matrix $A = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$ i) Find all the Eigen values and Eigen Vectors. ii) Verify that $S^{-1}AS$ is a diagonal matrix, where S is the matrix of Eigen vectors.		

Cont...

2	12 a)	Evaluate the integral $I = \int_0^1 \frac{dx}{1+x}$ using Gauss – Legendre Formula.	K3	CO2										
		(OR)												
	12 b)	Evaluate the integral $I = \int_{y=1}^{1.5} \int_{x=1}^2 \frac{dx dy}{x+y}$ using the Simpson's Rule with $h = 0.5$ (along $x - axis$) and $y = 0.25$ (along $y - axis$). Compare the Exact Solution.												
3	13 a)	Given $\sigma(\xi) = (23\xi^2 - 16\xi + 5)/12$, find $\rho(\xi)$ and write down an explicit linear multistep method.	K4	CO3										
		(OR)												
	13 b)	Solve the initial value problem $u' = -2tu^2, u(0) = 1$ with $h = 0.2$ on the interval $[0, 0.4]$, using P – C method $P : u_{j+1} = u_j + \frac{h}{2}(3u'_j - u'_{j-1})$ $C : u_{j+1} = u_j + \frac{h}{2}(u'_{j+1} + u'_j)$ as $P(EC)^m E, m = 2$												
4	14a)	Classify the following PDE: i) $2u_{xx} + 5u_{xy} + 2u_{yy} = 0$ ii) $4u_{xx} + 4u_{xy} + u_{yy} = 0$	K5	CO4										
		(OR)												
	b)	Solve $y'' = -y$ with boundary conditions $y(0) = 0, y(\pi) = 0$.												
5	15a)	Solve the Heat Equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ with step sizes $h = 1, k = 0.25$. Initial values: <table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>$u(x, 0)$</td><td>0</td><td>10</td><td>20</td><td>0</td></tr></table> Also find $u(1,1)$.	x	0	1	2	3	$u(x, 0)$	0	10	20	0	K6	CO5
	x	0	1	2	3									
	$u(x, 0)$	0	10	20	0									
	(OR)													
b)	Given that for Wave Equation <table><tr><td>x</td><td>0</td><td>1</td><td>2</td></tr><tr><td>$u(x, 0)$</td><td>0</td><td>10</td><td>0</td></tr><tr><td>$u(x, 1)$</td><td>0</td><td>8</td><td>0</td></tr></table> Find $u(1,2)$, given $r = 1$.	x	0	1	2	$u(x, 0)$	0	10	0	$u(x, 1)$	0	8	0	
x	0	1	2											
$u(x, 0)$	0	10	0											
$u(x, 1)$	0	8	0											

SECTION -C (30 Marks)

Answer ANY THREE questions

ALL questions carry EQUAL Marks

(3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO												
1	16	Find the largest Eigen value in modulus and the corresponding Eigen vector of the matrix $A = \begin{bmatrix} -15 & 4 & 3 \\ 10 & -12 & 6 \\ 20 & -4 & 2 \end{bmatrix}$ using Power Method.	K2	CO1												
2	17	Evaluate the integral $I = \int_1^2 \int_1^2 \frac{dx dy}{x+y}$ using Trapezoidal Rule with $h = k = 0.5$ and $h = k = 0.25$.	K3	CO2												
3	18	Solve the initial value problem $u'' = (1 + t^2)u, u(0) = 0, u'(0) = 0, t \in [0, 0.4]$ by Runge – Kutta Method with $h = 0.2$. Compare with exact solution $u(t) = e^{t^2/2}$.	K4	CO3												
4	19	Derive finite difference approximation for Laplace equation.	K5	CO4												
5	20	Solve the Heat Equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ for $0 \leq x \leq 4$. Given Boundary Conditions $u(0, t) = 0, u(4, t) = 0$ and Initial Conditions <table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>u (x, 0)</td><td>0</td><td>20</td><td>40</td><td>20</td><td>0</td></tr></table> Step sizes $h = 1, k = 0.25$. Find the solution at first two time levels.	x	0	1	2	3	4	u (x, 0)	0	20	40	20	0	K6	CO5
x	0	1	2	3	4											
u (x, 0)	0	20	40	20	0											

Z-Z-Z

END