(AUTONOMOUS)

MSc DEGREE EXAMINATION MAY 2026

(Third Semester)

Branch- **MATHEMATICS**

FUNCTIONAL ANALYSIS

Time: Three Hours   Maximum: 75 Marks

SECTION-A (10 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** marks $(10 \times 1 = 10)$

Module No.	Question No.	Question	K Level	CO
1	1	Name the subspace Y of a Banach Space X where the set Y is closed in X. a) unbounded b) open c) complete d) continuous	K1	CO1
	2	Relate the statement "the every finite dimensional subspace Y of a normed space X" with the following. a) open b) closed c) bounded d) not bounded	K2	CO1
2	3	What is the condition for the orthogonal element of the inner product space X where $x, y \in X$. a) $\langle x,y\rangle = 0$ b) $\langle x,y\rangle = 1$ c) $\langle x,y\rangle \neq 1$ d) $\langle x,y\rangle \neq 0$	K1	CO2
	4	Relate Y when Y is a closed subspace of a Hilbert Space H. a) 0 b) $(Y)^{-1}$ c) $Y^{\perp}$ d) $Y^{\perp\perp}$	K2	CO2
3	5	How many minimal elements are there in a partially ordered set $M \neq \emptyset$ when every chain $C \subset M$ has an upper bound?. a) at least one b) at least two c) zero d) at most one	K1	CO3
	6	Relate $\langle T^{*}y, x\rangle$ when $T: H_1 \to H_2$ is the bounded linear operator for the Hilbert Spaces H_1 and H_2. a) $\langle Ty, x\rangle$ b) $\langle x, Ty\rangle$ c) $\langle y, Tx\rangle$ d) $\langle T^{*}x, y\rangle$	K2	CO3
4	7	Which of the following is true for a bounded linear operator T from a Banach space X onto a Banach space Y and T is a open mapping and bijective? a) T^{-1} is discontinuous b) T^{-1} is continuous c) T^{-1} is differentiable d) T^{-1} is not differentiable	K1	CO4
	8	Interpret the following condition with "X is embeddable in a normed space Z". a) X is subset of Z b) X is quotient of Z c) X is homeomorphic of Z d) X is isomorphic with a subspace of Z	K2	CO4
5	9	Which of the following is true for a contraction $T: X \to X$ on a metric space? a) discontinuous b) differentiable c) continuous d) partially differentiable	K1	CO4
	10	Interpret the condition for $x \in X$ to be a fixed point for a mapping $T: X \to X$? a) $Tx = x$ b) $Tx = 1$ c) $Tx \neq x$ d) $Tx = 0$	K2	CO4

Cont...

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	Show that every finite dimensional subspace Y of a normed space X is complete.	K2	CO1
		(OR)		
	11.b.	Show that every linear operator on X is bounded when a normed space X is finite dimensional.		
2	12.a.	Construct the proof for "The space C[a,b] is not an inner product space, hence not a Hilbert space".	K3	CO2
		(OR)		
	12.b.	Construct the proof for "For any subset $M \neq \emptyset$ of a Hilbert Space H, the span of M is dense in H if and only if $M^{\perp} = \{0\}$".		
3	13.a.	Test the following condition i. $Q = 0$ if and only if $\langle Qx, y\rangle = 0$ for all $x \in X$ and $y \in Y$. ii. If $Q: X \to X$ where X is complex, and $\langle Qx, x\rangle = 0$ for all $x \in X$, then $Q = 0$. Where X and Y are inner product spaces and $Q: X \to Y$ a bounded linear operator.	K3	CO3
		(OR)		
	13.b.	Construct the proof for statement "Every vector space $X \neq \{0\}$ has a Hamel basis."		
4	14.a.	Prove that "Every Hilbert space H is reflexive".	K4	CO4
		(OR)		
	14.b.	Test the condition "The sequence of norms $\|\|T_n\|\|$ is bounded", where (T_n) be a sequence of bounded linear operators $T_n: X \to Y$ from a Banach Space X into a normed space Y such that $(\|\|T_n x\|\|)$ is bounded for every $x \in X$.		
5	15.a.	Discover the proof for "T is compact if and only if it maps every bounded sequence (x_n) in X onto a sequence (Tx_n) in Y which has a convergent subsequence", where X and Y be normed spaces and $T: X \to Y$ a linear operator.	K4	CO4
		(OR)		
	15.b.	Discover the proof for "A compact linear operator $T: X \to Y$ from a normed space X into a Banach space Y has a compact linear extension $\tilde{T}: \hat{X} \to Y$, where is $\hat{x}$ the completion of X".		

SECTION -C (30 Marks)

Answer **ANY THREE** questions

ALL questions carry **EQUAL** Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	Discover the proof for "If Y is a Banach space, then $B(X, Y)$ is a Banach space".	K4	CO1
2	17	State and prove the Schwarz inequality and Triangle inequality.	K4	CO1
3	18	Write the statement and proof for the Hahn-Banach (Extension of linear functionals) theorem.	K4	CO2
4	19	Discover the proof for the statement "A bounded linear operator T from a Banach space X onto a Banach space Y has the property that the image $T(B_0)$ of the open unit ball $B_0 = B(0; 1) \subset X$ contains an open ball about $0 \in Y$.	K4	CO4
5	20	State and prove Banach fixed point theorem (Contraction Theorem).	K4	CO4

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