(AUTONOMOUS)

MSc DEGREE EXAMINATION MAY 2026

(Fourth Semester)

Branch – **MATHEMATICS**

CONTROL THEORY

Time: Three Hours

Maximum: 75 Marks

SECTION-A (10 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** marks (10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	If X is a Banach space and $T: X \to X$ is a contraction mapping, then T has a _____ (a) unique fixed point (b) couple fixed point (c) proximal point (d) dual point	K1	CO1
	2	The $n \times r$ matrix function $R(t)$ defined on $[0, T]$ is an (initial state) reconstruction kernel if and only if (a) $\int_0^\infty R(t)H(t)X(t,0)dt = I$ (b) $\int_0^T R(t)H(t)X(t,0)dt = O$ (c) $\int_0^\infty R(t)H(t)X(t,0)dt = I$ (d) $\int_0^T R(t)H(t)X(t,0)dt = I$	K2	CO1
2	3	The constant coefficient control system $\dot{x} = Ax + Bu$ is controllable if and only if rank $[B, AB, \dots A^{n-1}B] =$ (a) 1 (b) 0 (c) n (d) ∞	K1	CO2
	4	An $m \times n$ matrix function $S(t)$ with entries in $L_m^2[0,T]$ is a streering function for $\dot{x} = A(t)x + B(t)u$ on $[0,T]$ if (a) $\int_0^\infty R(t)H(t)X(t,0)dt = I$ (b) $\int_0^T X(T,t)B(t)S(t)dt = I$ (c) $\int_0^\infty X(T,t)B(t)S(t)dt = I$ (d) $\int_0^T R(t)H(t)X(t,0)dt = I$	K2	CO2
3	5	The solution of $\phi(t)$ is called _____ if it is not stable (a) un stable (b) complete stable (c) strong stable (d) uniform stable	K1	CO3
	6	Eigen values of the matrix $\begin{bmatrix} 1 & 5 \\ 5 & 1 \end{bmatrix}$ are (a) $-4,6$ (b) $-4,-6$ (c) $-4,0$ (d) 1,5	K2	CO3
4	7	The Modified system $\dot{x} = (A + BK)x$ is called an ______ (a) open loop system (b) closed loop system (c) circuit loop system (d) closed circuit loop system	K1	CO4
	8	$C(A, B)$ is invariant subspace of the matrix ___ (a) O (b) I (c) A (d) B	K2	CO4
5	9	$u(t) = G(t)x(t), t \in [0,T]$, where $G(t)$ is an ___ (a) $n \times n$ matrix valued function (b) $1 \times n$ matrix valued function (c) $m \times 1$ matrix valued function (d) $m \times n$ matrix valued function	K1	CO5
	10	In the cost functional equation, $R(t)$ is ___ matrix (a) an $m \times n$ symmetric positive semidefinite (b) an $n \times n$ symmetric positive semidefinite (c) an $n \times m$ symmetric positive semidefinite (d) an $m \times m$ symmetric positive definite	K2	CO5

Cont...

SECTION - B (35 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	Let X be a Banach space and let $T: X \to X$ be such that $T^n: X \to X$ is a contraction for some n. Then show that T has a unique fixed point.	K2	CO1
		(OR)		
	11.b.	Let $x_1, x_2, \dots x_k$ be vectors in R^n and let $x_1(t), x_2(t), \dots x_k(t)$ be the solutions of $\dot{x} = A(t)x$ on $[0,T]$ with $x_i(0) = x_i, i = 1,2, \dots k$. Let y_i be observations on $[0,T]$ defined by $y_i(t) + H(t)x_i(t), t \in [0,T]$. Then show that the observed linear system $\dot{x} = A(t)x, y(t) = H(t)x(t)$ is observable on $[0,T]$ if and only if y_i are linearly independent in $L_r^2[0,T]$ whenever the x_i are linearly independent in R^n		
2	12.a.	Prove that the system $\dot{x} = A(t)x + B(t)u$ is controllable on $[0,T]$ if and only if the "controllability Grammian" $M(0,T) = \int_0^T X(T,t)B(t)B^*(t)X^*(T,t)\,dt$ is positive definite.	K3	CO2
		(OR)		
	12.b.	Assume that the continuous function f satisfies the condition $\lim\limits_{\lvert(x,u)\rvert\to\infty} \frac{\lvert f(t,x,u)\rvert}{\lvert(x,u)\rvert} = 0$ uniformly for $t \in I$. If the system $\dot{x} = A(t)x + B(t)u$ is completely controllable, then prove that system $\dot{x}(t) = A(t)x(t) + B(t)u(t) + f(t, x(t), u(t))$is completely controllable.		
3	13.a.	If all the characteristic roots of A have negative real parts and $B(t)$ satisfies the condition $\lim\limits_{t\to\infty} \lVert B(t)\rVert = 0$ then solve all the solutions of the system $\dot{x}(t) = Ax + B(t)x$ tend to zero as $t \to \infty$	K3	CO3
		(OR)		
	13.b.	Suppose that the function $g(x)/\lVert x\rVert$ is a continuous function of x which tends to zero for $x = 0$. Then prove that the solution $x(t) = 0$ of $\dot{x}(t) = Ax + g(x)$ is asymptotically stable if the solution $x(t) = 0$ of the linearized equation $\dot{x} = Ax$ is asymptotically stable.		
4	14.a.	Show that the pair $(A + BK, B)$ is controllable if and only if the pair (A, B) is controllable.	K4	CO4
		(OR)		
	14.b.	Prove that the pair (H, A) is detectable if and only if the pair $(A^*, -H^*)$ is stabilizable.		
5	15.a.	If the eigenvalues of the matrix G have negative real parts, then prove that inspect that the optimal system $\dot{x}(t) = Gx(t)$ is stable.	K4	CO5
		(OR)		
	15.b.	If $x(t)$ and $p(t)$ are the solutions of the canonical equations $\begin{bmatrix}\dot{x}(t)\\ \dot{p}(t)\end{bmatrix} = \begin{bmatrix}A(t) & -S(t)\\ -Q(t) & -A^*(t)\end{bmatrix}\begin{bmatrix}x(t)\\ p(t)\end{bmatrix}$ and if $p(t) = K(t)x(t)$ for all $t \in [0,T]$ and all $x(t)$, prove that the $K(t)$ must satisfies the equation $\dot{K}(t) + K(t)A(t) + A^*(t)K(t) - K(t)S(t)K(t) + Q(t) = 0$		

Cont...

Answer **ANY THREE** questions

ALL questions carry **EQUAL** Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	Solve the initial value problem $$\dot{x}(t) = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & -2 \\ 3 & 2 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ e^t \cos 2t \end{bmatrix},$$ $$x(0) = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$ where $x(t) = \left(x_1(t), x_2(t), x_3(t)\right)^*$.	K4	CO1
2	17	Prove that the system is controllable on $[0, T]$ if and only if the adjoint linear observed system $$\dot{y} = -A^*(t)y$$ $$w = B^*(t)y$$ Is observable on $[0, T]$	K4	CO2
3	18	Let $X(t)$ be a fundamental matrix of the system $\dot{x}(t) = A(t)x(t)$. Assume further that there exists a constant $K > 0$ such that $$\int_0^t \lVert X(t,x) \rVert ds \le K, t \ge 0$$ Then prove thatthere exists a constant $M > 0$ such that $$\lVert X(t) \rVert \le Me^{-\left(\frac{1}{K}\right)t}, t \ge 0$$	K4	CO3
4	19	Show that the time invariant system is controllable if and only if there exist $m \times n$ matrices K_1, K_2 for which the matrix $I - e^{-(A+BK_2)T} e^{(A+BK_1)T}$ is invertible	K4	CO4
5	20	Prove that for the continuous non linear system $\dot{x}(t) = A(t)x(t) + B(t)u(t) + f\left(t, x(t)\right)$ with quadratic performance criteria $J = \frac{1}{2}x^*(T)Fx(T)$ $+\frac{1}{2}\int_0^T [x^*(t)Q(t)x(t) + u^*(t)R(t)u(t)]dt$ the optimal control exists if $\lVert f(t,x) - f(t,y) \rVert \le a\lVert x - y \rVert$ where a is positive constant, and is given by $$u(x(t), t) = -R^{-1}(t)B^*(t)K(t)x(t) - R^{-1}(t)B^*(t)h(t,x)$$ Where $K(t)$ satisfies the Riccati equation and $$h(t,x) = -[A^*(t) - K(t)B(t)R^{-1}(t)B^*(t)]h(t,x) - K(t)f(t,x(t))$$ $$h(T,x) = 0$$	K4	CO5

Z-Z-Z END