(Second Semester)

Branch - **MATHEMATICS**

TOPOLOGY

Time: Three Hours | Maximum: 75 Marks

SECTION-A (10 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** marks (10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	The basis for the order topology on a simply ordered set X consists of sets of which types? (i) Open intervals (a, b) only (ii) Open intervals (a, b), plus $[a_0, b)$ if a_0 is the smallest element, and $(a, b_0]$ if b_0 is the largest element (iii) All closed intervals [a, b] (iv) Only open rays $(a, +\infty)$ and $(-\infty, b)$	K1	CO1
	2	A subbasis S for a topology on X requires that (i) S covers X and is closed under finite intersections (ii) The union of elements of S equals X (iii) S is already a topology (iv) Every element of S is an open interval	K2	CO1
2	3	Which of the following correctly explains why a constant function $f(x) = y_0$ is continuous? (i) f is bijective (ii) $f^{-1}(V)$ is either X or $\emptyset$, both of which are open (iii) f maps open sets to open sets (iv) f is an imbedding	K1	CO2
	4	The square metric ρ on $\mathbb{R}^n$ is defined as (i) $\rho(x,y) = \Sigma\lvert x_i - y_i\rvert$ (ii) $\rho(x,y) = \min\{\lvert x_i - y_i\rvert\}$ (iii) $\rho(x,y) = (\Sigma(x_i - y_i)^2)^{(1/2)}$ (iv) $\rho(x,y) = \max\{\lvert x_i - y_i\rvert\}$	K2	CO2
3	5	If $\{A_\alpha\}$ is a collection of connected subspaces of X with a common point p, then $\cup A\alpha$ is (i) Compact (ii) Disconnected (iii) Connected (iv) Open	K1	CO3
	6	The components of a topological space X are (i) The open subsets of X (ii) Maximal connected subspaces of X (iii) Minimal connected subspaces of X (iv) Always open sets	K2	CO3
4	7	Compactness always implies limit point compactness. The converse (i) is always true (ii) is true only for Hausdorff spaces (iii) is true for metrizable spaces but not in general (iv) is never true	K1	CO4
	8	The lower limit topology $\mathbb{R}$ satisfies (i) The second countability axiom (ii) The first countability axiom and is separable, but not second-countable (iii) None of the countability axioms (iv) All countability axioms	K2	CO4
5	9	Which of the following is always normal? (i) Every Hausdorff space (ii) Every regular space (iii) Every compact Hausdorff space (iv) Every product of normal spaces	K1	CO5
	10	The Sorgenfrey plane $\mathcal{R}_l^{\,2}$ is an example of a space that is (i) Regular but not normal (ii) Normal (iii) Not even Hausdorff (iv) Compact	K2	CO5

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Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	Let X be a topological space. Suppose that $\mathcal{C}$ is a collection of open sets of X such that for each open set U of X and each x in U, there is an element C of $\mathcal{C}$ such that $x \in C \subset U$. Show that $\mathcal{C}$ is a basis for the topology of X.	K3	CO1
		(OR)		
	11.b.	If A is a subspace of X and B is a subspace of Y, then show that the product topology on $A \times B$ is the same as the topology $A \times B$ inherits as a subspace of $X \times Y$.		
2	12.a.	Show that finite point set in a Hausdorff space X is closed.	K3	CO2
		(OR)		
	12.b.	Let $f: X \to Y$. If f is continuous, show that for every convergent sequence $x_n \to x$ in X, the sequence $f(x_n)$ converges to $f(x)$. Also show that the converse holds if X is metrizable		
3	13.a.	State and prove Intermediate value theorem.	K4	CO3
		(OR)		
	13.b.	Show that every closed subspace of a compact space is compact.		
4	14.a.	Let X be locally compact Hausdorff; let A be a subspace of X. If A is closed in X or open in X, show that A is locally compact.	K4	CO4
		(OR)		
	14.b.	Suppose X has a countable basis. Show that (a) Every open covering of X contains a countable subcollection covering X (b) There exists a countable subset of X that is dense in X.		
5	15.a.	Show that every metrizable space is normal.	K5	CO5
		(OR)		
	15.b.	Show that every regular space with a countable basis is normal.		

SECTION -C (30 Marks)

Answer **ANY THREE** questions

ALL questions carry **EQUAL** Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	Let A be a subset of the topological space X. Prove the following (a) $x \in \bar{A}$ if and only if every open set U containing x intersects A. (b) If the topology of X is given by a basis, then $x \in \bar{A}$ if and only if every basis element B containing x intersects A.	K3	CO1
2	17	Show that the topologies on $\mathbb{R}^n$ induced by the Euclidean metric d and the square metric ρ. are the same as the product topology on $\mathbb{R}^n$.	K3	CO2
3	18	Show that the product of finitely many compact spaces is compact.	K4	CO3
4	19	Show that (a) a subspace of a Hausdorff space is Hausdorff and a product of Hausdorff space is Hausdorff. (b) a subspace of a regular space is regular and a product of regular space is regular.	K5	CO4
5	20	State and prove Tietze extension theorem.	K4	CO5

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