(First Semester)

Branch - **MATHEMATICS**

ORDINARY DIFFERENTIAL EQUATIONS

Time: Three Hours   Maximum: 75 Marks

SECTION-A (10 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** marks (10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	The equation $y'' + \omega^2 y=0$ is important in the study of oscillatory behavior in many physical situations, and is called the _______ equation. a) Harmonic oscillator b) Homogeneous c) Non-Harmonic oscillator d) Non –Homogeneous	K1	CO1
	2	Two functions φ_1, φ_2 defined on an interval I are said to be __________ on I if there exist two constants c_1, c_2, not both zero, such that $c_1\varphi_1(x) + c_2\varphi_2(x)=0$ for all x in I. a) Linearly independent b) Non linearly dependent c) Linearly dependent d) Non linearly dependent	K2	CO1
2	3	If $\varphi_1 \ldots\ldots, \varphi_n$ are n solution of $L(y) = 0$ on an interval I, they are _____ there iff, $W(\varphi_1 \ldots\ldots, \varphi_n)(x) \neq 0$ for all x in I. a) Linearly independent b) Non linearly dependent c) Linearly dependent d) Non linearly independent	K1	CO2
	4	Let $\emptyset_1 \ldots\ldots, \emptyset_n$ are n solution of L(y)=0 on an interval I containing x_0. then they are linearly independent on I iff _______for all x in I. a) $W(\emptyset_1 \ldots\ldots, \emptyset_{n-1})(x_0)=0$ b) $W(\emptyset_1 \ldots\ldots, \emptyset_n)(x_0) \neq 0$ c) $W(\emptyset_1 \ldots\ldots, \emptyset_n)(x_0)=0$ d) $W(\emptyset_1 \ldots\ldots, \emptyset_{n-1})(x_0) \neq 0$	K2	CO2
3	5	Let $\emptyset_1 \ldots\ldots, \emptyset_n$ be n solutions of L(y)=0 on an interval I, and let x_0 be any point I, then $W(\emptyset_1 \ldots\ldots, \emptyset_n)(x) =$ _________ a) Exp $[\int_{x_0}^{x} a_1(t)dt]$ $W(\emptyset_1 \ldots\ldots, \emptyset_n)(x_0)$ b) Exp $[-\int_{x_0}^{x} a_1(t)dt]$ $W(\emptyset_1 \ldots\ldots, \emptyset_{n-1})(x_0)$ c) Exp $[\int_{x_0}^{x_n} a_1(t)dt]$ $W(\emptyset_1 \ldots\ldots, \emptyset_n)(x_0)$ d) Exp $[-\int_{x_0}^{x} a_1(t)dt]$ $W(\emptyset_1 \ldots\ldots, \emptyset_n)(x_0)$	K1	CO3
	6	If g is a function defined on an interval I containing a point x_0 *that g is* _____ at x_0. a) Homogeneous b) Non homogeneous c) Analytic d) Non Analytic	K2	CO3
4	7	Suppose k is a root of q of multiplicity one then there is a solution ψ of $L(y)=x^k$ of the form ______________ a) $\psi(x_0) = -cx^k \log x$ b) $\psi(x) = -cx^k \log x$ c) $\psi(x) = cx^k \log x$ d) $\psi(x) = cx^k \log(-x)$	K1	CO4
	8	The regular singular points for the legendre equation are ___________ a) $(1-x^2)y'' - 2xy' + \alpha(\alpha+1)y = 0$ b) $(1-x^2)y'' + 2xy' + \alpha(\alpha+1)y = 0$ c) $(1-x^2)y'' - 2xy' + \alpha(\alpha-1)y = 0$ d) $(1-x^2)y'' + 2xy' + \alpha(\alpha-1)y = 0$	K2	CO4
5	9	Let f satisfies a _____________on S if there exists a constant K>0 such that $\lvert f(x,y_1) - f(x,y_2)\rvert \leqq K\lvert y_1 - y_2\rvert$ for all $(x,y_1),(x,y_2)$ in S. a) Singular condition b) Lipschitz condition c) Oscillator equation d) Bessel equation	K1	CO5
	10	The k[th] successive approximation $\emptyset_k$ to the solution $\emptyset$ of the initial value problem of existence theorem satisfies __________ for all x in I. a) $\lvert\emptyset(x) - \emptyset_k(x)\rvert \leqq \frac{M}{K}\frac{(K\alpha)^{k+1}}{k+1}e^{k\alpha}$ b) $\lvert\emptyset(x) + \emptyset_k(x)\rvert \leqq \frac{M}{K}\frac{(K\alpha)^{k+1}}{k+1}e^{k\alpha}$ c) $\lvert\emptyset(x) - \emptyset_k(x)\rvert \leqq \frac{M}{K}\frac{(K\alpha)^{k-1}}{k+1}e^{k\alpha}$ d) $\lvert\emptyset(x) - \emptyset_k(x)\rvert \leqq \frac{M}{K}\frac{(K\alpha)^{k+1}}{(k+1)!}e^{k\alpha}$	K2	CO5

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ALL questions carry **EQUAL** Marks (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	Prove that Let $\emptyset_1, \emptyset_2$ e the two solutions of L(y)=0 given by ($\emptyset_1(x) = e^{r_1 x}, \emptyset_2(x) = e^{r_2 x}$) in case $r_1 \neq r_2$, and by ($\emptyset_1(x) = e^{r_1 x}, \emptyset_2(x) = e^{r_1 x}$) in case $r_1 = r_2$. If c_1, c_2 are any two constants the function $\emptyset$=$c_1\emptyset_1 + c_2\emptyset_2$ is a solution of L(y)=0 on $-\infty < x < \infty$. Conversely, if $\emptyset$ is any solution of L(y)=0 on $-\infty < x < \infty$, there are unique constants c_1, c_2 such that $\emptyset$=$c_1\emptyset_1 + c_2\emptyset_2$.	K2	CO1
		(OR)		
	11.b.	Let $\emptyset_1, \emptyset_2$ be any two linearly independent solutions of L(y)=0 on an interval I. Prove that Every solution $\emptyset$ of L(y)=0 can be written uniquely as $\emptyset$=$c_1\emptyset_1 + c_2\emptyset_2$ where c_1, c_2 *are constants.*		
2	12.a.	Let $\emptyset_1$,……., $\emptyset_n$ be n linearly independent solutions of L(y)=0 on an interval I. If c_1,…… c_n are any constants $\emptyset = c_1\emptyset_1$+…….+$c_n\emptyset_n$ is a solution, and every solution may be represented in this form -prove it.	K2	CO2
		(OR)		
	12.b.	Let b be continuous on an interval I, and let $\emptyset_1$,……., $\emptyset_n$ be n linearly independent solutions of L(y)=0 on I. Every solution ψ of L(y)=b(x) can be written as ψ=ψ_p+$c_1\emptyset_1$+…….+$c_n\emptyset_n$, where ψ_p is a particular solution of L(y)=b(x), and c_1,…… c_n are constants. Every such ψ is a solution of L(y)=b(x). A particular solution ψ_p is given by $\psi_p(x)=\sum_{k=1}^{n} \emptyset_k(x) \int_{x_0}^{x} \frac{W_k(t)b(t)}{W(\emptyset_1,\ldots,\emptyset_n)(t)}\, dt$. Prove it		
3	13.a.	State and prove Uniqueness theorem.	K3	CO3
		(OR)		
	13.b.	If $\emptyset_1$ is a solution of *L(y)=y"+$a_1(x)y' + a_2(x)y$ =0 on an interval I, and* $\emptyset_1(x) \neq 0$ *on I, a second solution* $\emptyset_2$ *of* $\emptyset_2(x) = \emptyset_1(x) \int_{x_0}^{x} \frac{1}{[\emptyset_1(s)]^2} exp\left(-\int_{x_0}^{s} a_1(t)dt\right)ds$. *Prove that The function* $\emptyset_1, \emptyset_2$ *form a basis for the solution of L(y)=y"+$a_1(x)y' + a_2(x)y$ =0 on I.*		
4	14.a.	Compute solution for the Bessel equation of order α for the first kind.	K3	CO4
		(OR)		
	14.b.	Compute the indicial polynomials, and their roots $4x^2y'' + (4x^4 - 5x)y' + (x^2 + 2)y = 0$		
5	15.a.	Suppose the equation M(x,y)+N(x,y)y'=0----(1) is exact in a rectangle R, and F is a real-valued function such that $\frac{dF}{dx} = M$, $\frac{dF}{dx} = N$---(2) in R. Prove that Every differential function φ defined implicitly by a relation $F(x, y) = c$, is a solution of (1) and every solution of (1) whose graph lies in R.	K3	CO5
		(OR)		
	15.b.	Prove that a function $\emptyset$ is a solution of the initial value problem y'=f(x,y), y(x_0)= y_0 on an interval I if and only if it is a solution of the integral equation y=y_0 + $\int_{x_0}^{x} f(t, y)\, dt$ on I.		

SECTION -C (30 Marks)

Answer **ANY THREE** questions

ALL questions carry **EQUAL** Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	Let $\emptyset$ be any solution of L(y)= y''+$a_1y' + a_2y$ =0 On an interval I containing a point x_0 . Then prove that for all x inI, $\|\emptyset(x_0)\|e^{-k\|x-x_0\|} \leqq \|\emptyset(x)\| \leqq \|\emptyset(x_0)\|e^{k\|x-x_0\|}$ where $\|\emptyset(x)\|=[\|\emptyset(x)\|^2 + \|\emptyset'(x)\|^2]^{\frac{1}{2}}$, k=1+$\|a_1\| + \|a_2\|$.	K3	CO1
2	17	Consider the equation with constant coefficients L(y)=P(x)e^{ax}, where P is the polynomial given by P(x)=$b_0x^m + b_1x^{m-1}$+………..+b_m , $b_0 \neq 0$.Suppose a is a root of the characteristic polynomial p of L of multiplicity j. Then prove that there is a unique solution $\psi(x) = x^j(c_0x^m + c_1x^{m-1} + \cdots + c_m)e^{ax}$, where $c_0, c_1, \ldots, c_m$ are constants determine by the annihilator method.	K4	CO2
3	18	Let $\emptyset_1$,……., $\emptyset_n$ be n linearly independent solutions of L(y)=0 on an interval I. If $\emptyset$ is any solution of L(y)=0 on I, it can be represented in the form $\emptyset$=$c_1\emptyset_1$+… +$c_n\emptyset_n$, where c_1,……. c_n are constants. Prove that any set of n linearly independent solutions of L(y)=0 on I is a basis for the solutions of L(y)=0 on I.	K4	CO3
4	19	Derive Bessel equation.	K4	CO4
5	20	State and Prove Existence theorem.	K3	CO5

Z-Z-Z END