(AUTONOMOUS)

MSc DEGREE EXAMINATION MAY 2026

(Fourth Semester)

Branch – **MATHEMATICS**

OPERATOR THEORY

Time: Three Hours | Maximum: 75 Marks

SECTION-A (10 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** marks (10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	Which of the following is an isometry operator? a) $T^* = T$ b) $T^*T = TT^*$ c) $T^*T = I$ d) $T^2 = T$	K1	CO1
	2	If T is an operator on a Hilbert space H over the complex scalars C, then T is normal iff _____ for all $x \in H$ a) (Tx, x) is real b) $\|\|Tx\|\| = \|\|T^*x\|\|$ c) $\|\|Tx\|\| = \|\|T^*x\|\| = \|\|x\|\|$ d) $\|\|Tx\|\| \geq \|\|T^*x\|\|$	K2	CO1
2	3	An operator U on a Hilbert space H is a unitary operator if and only if _____ a) $U^* = U$ b) $U^*U = UU^* = I$ c) $U^2U = I$ d) $U^2 = U$	K1	CO2
	4	When an operator T is doubly commutes with S? a) T commutes with S b) T commutes with S* c) T not commutes with S and S* d) T commutes with both S and S*	K2	CO2
3	5	If T is a self-adjoint operator on a Hilbert space H, then all the eigen values of T are _____ numbers a) real b) complex c) rational d) irrational	K2	CO3
	6	An operator T is spectraloid if _____ a) $\omega(T) \neq r(T)$ b) $\omega(T) \geq r(T)$ c) $\omega(T) \leq r(T)$ d) $\omega(T) = r(T)$	K1	CO3
4	7	If T is a hyponormal operator, then which of the following is incorrect? a) $T + \mu$ is also a hyponormal for any $\mu \in C$ b) T is a transaloid operator c) T^{-1} is also a hyponormal operator if T^{-1} exists d) T is a condition G_1 operator	K2	CO4
	8	Pick out the correct form a) Paranormal $\supseteq$ Normaloid b) Normal $\supseteq$ Quasinormal c) Subnormal $\subseteq$ Hyponormal d) Spectraloid $\subseteq$ Normaloid	K2	CO4
5	9	$A^\alpha \geq B^\alpha$ does not hold in general for any $\alpha > 1$, even if _____ a) $A \leq B \leq 0$ b) $A \geq B \geq 0$ c) $A \leq 0 \leq B$ d) $B \leq 0 \leq A$	K1	CO5
	10	If T is a positive invertible operator on a Hilbert space H and $\lambda > 1$, then _____ a) $\lambda T + (1-\lambda) \leq T^\lambda$ b) $\lambda T + (1-\lambda) \geq T^\lambda$ c) $\lambda T + (1-\lambda) < T^\lambda$ d) $\lambda T + (1-\lambda) > T^\lambda$	K1	CO5

Cont...

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	Let T be an operator on a Hilbert space H. Then demonstrate that T* is also an operator on H, and the following properties hold (i) $\|T^*\| = \|T\|$ (ii) $(T_1 + T_2)^* = T_1^* + T_2^*$ (iii) $(\alpha T)^* = \overline{\alpha} T^*$ for any $\alpha \in C$ (iv) $(T^*)^* = T$ (v) $(ST)^* = T^*S^*$	K3	CO1
		(OR)		
	11.b.	Let P_1 and P_2 be two projections onto M_1 and M_2 respectively. Then prove that (i) $P = P_1 + P_2$ is a projection if and only if $M_1 \perp M_2$. (ii) If $P = P_1 + P_2$ is a projection, then P is the projection onto $M_1 \oplus M_2$.		
2	12.a.	Construct the conditions when an operator U on a Hilbert space H is (i) an isometry operator and (ii) a unitary operator	K3	CO2
		(OR)		
	12.b.	Let T be an operator on a Hilbert space H and M be a closed subspace of H. Then determine that the following conditions are mutually equivalent. (i) M reduces T (ii) $M^\perp$ reduces T (iii) M reduces T* (iv) M is invariant under T and T* (v) TP = PT where P is the projection onto M.		
3	13.a.	If an operator T is normal, then $\sigma(T) = A_o(T)$ holds. Justify this statement.	K4	CO3
		(OR)		
	13.b.	Examine the proof for the different characterizations of spectraloid operators.		
4	14.a.	If T is a paranormal operator, then analyze the following properties (i) T^n is also paranormal for any natural number n (ii) T is normaloid operator, $\|T\| = r(T)$ (iii) If T is an invertible paranormal operator, so is T^{-1}	K4	CO4
		(OR)		
	14.b.	Confirm that every hyponormal and normal operator is convexoid.		
5	15.a.	If T is a positive invertible operator on a Hilbert space H, then reveal that the following hold and are mutually equivalent: (i) If $1 \geq \lambda \geq 0$, then $\lambda T + (1-\lambda) \geq T^\lambda$ (ii) If $\lambda > 1$, then $\lambda T + (1-\lambda) \leq T^\lambda$ (iii) If $\lambda < 0$, then $\lambda T + (1-\lambda) \leq T^\lambda$	K5	CO5
		(OR)		
	15.b.	Formulate Lowner-Heinz inequality and provide its proof.		

SECTION -C (30 Marks)

Answer **ANY THREE** questions

ALL questions carry **EQUAL** Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	For any positive operator A, there exists the unique positive operator S such that $S^2 = A$ and $(S) \supset (A)$ – Validate this statement.	K4	CO1
2	17	If U is an operator on a Hilbert space H, then prove that the following are mutually equivalent (i) U and U* is a partial isometry operator (ii) $UU^*U = U$ and $U^*UU^* = U^*$ (iii) U^*U and UU^* is a projection operator	K4	CO2
3	18	Discuss about the power inequality of $\omega(T)$	K4	CO3
4	19	Prove that an operator T is convexoid if and only if (i) $T - \lambda$ is spectraloid for all complex number λ (ii) $\|(T-\mu)^{-1}\| \leq \dfrac{1}{d(\mu, co\sigma(T))}$ for all $\mu \notin co\sigma(T)$	K5	CO4
5	20	By providing logical arguments, prove Furuta inquality.	K5	CO5

Z-Z-Z END