MSc DEGREE EXAMINATION MAY 2026

(Second Semester)

Branch - **MATHEMATICS**

MEASURE THEORY

Time: Three Hours Maximum: 75 Marks

SECTION-A (10 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** marks (10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	What is the Lebesgue Measure for the countable set of real numbers? (i) Infinite (ii) One (iii) Zero (iv) Undefined	K1	CO1
	2	Lebesgue outer measure is (i) Sub additive (ii) Countably Additive (iii) Translation Variant (iv) Not defined for open sets	K2	CO1
2	3	If $f(x) \geq 0$ is a measurable function then and $\int f d\mu = 0$ then (i) $f(x) = 0$ for all x (ii) $f(x) = 0$ almost everywhere (iii) $f(x) > 0$ almost everywhere (iv) The domain of f is a set of measure zero	K1	CO2
	4	A function f is Lebesgue integrable if (i) $\int f d\mu = 0$ (ii) $\int f d\mu = \infty$ (iii) $\int \lvert f \rvert d\mu < \infty$ (iv) $\int f^+ d\mu < \infty$ only	K2	CO2
3	5	A measure space (X, F, μ) is called σ – finite if (i) X is finite (ii) $\mu(X) < \infty$ (iii) X can be written as countable union of sets with finite measure (iv) $\mu(E) = 0$, for all $E \in$ F	K1	CO3
	6	Which of the following is true about the outer measure of a countable set? (i) It is always ∞ (ii) It is 0 (iii) It is 1 (iv) Not defined	K2	CO3
4	7	Which of the following functions is concave? (i) x^2 (ii) e^x (iii) $logx$ (iv) x^p, $p > 1$	K1	CO4
	8	Minkowski's inequality is actually a statement that: (i) L^p is a complete space (ii) The L^p norm satisfies the triangle inequality (iii) L^p is separable (iv) Holder's inequality is valid	K2	CO4
5	9	In the Hahn Decomposition X = PUN for a signed measure μ, the set P is called (i) Negative set (ii) Positive set (iii) Null set (iv) sigma – finite set	K1	CO5
	10	*A set E* is called a v – null set if (i) v(E) =∞ (ii) v(F) = 0for all measurable F⊂ E (iii) v(F) ≥ 0for all measurable F⊂ E (iv) v (E) <,0	K2	CO5

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SECTION - B (35 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	Prove that if $\{E_n\}$ is a sequence disjoint of measurable sets, then $m(\cup En) = \sum m(En)$	K2	CO2
		(OR)		
	11.b.	Prove that if f and g are measurable , f+g, f.g and max (f,g) are also measurable.		
2	12.a.	Prove that for non-negative measurable functions f and g, $\int af + bg = a\int f + b\int g$, a,b > 0	K1	CO2
		(OR)		
	12.b.	Prove that if $\{f_n\}$ is a sequence of non negative measurable functions, then $\int \sum_{n=1}^{\infty} fn = \sum_{n=1}^{\infty} \int fn$		
3	13.a.	Show that K(R) = $\{E;\ E \subseteq \bigcup_{n=1}^{\infty} E_n, E_n \in R\}$	K4	CO3
		(OR)		
	13.b.	If μ is a σ – finite measure on R, Show that the extension of $\bar{\mu}$ of μ to δ^* is also σ – finite.		
4	14.a.	State and prove Minkowski's Inequality.	K5	CO4
		(OR)		
	14.b.	Prove that the space $L^p(\mu)$ is a complete metric space for $1 \leq p \leq \infty$.		
5	15.a.	Show that $\left\lvert\frac{d\vartheta}{d\mu}\right\rvert = \frac{d\lvert\vartheta\rvert}{d\mu}$, μ almost everywhere	K2	CO1
		(OR)		
	15.b.	Prove the Hahn Decomposition Theorem.		

SECTION -C (30 Marks)

Answer **ANY THREE** questions

ALL questions carry **EQUAL** Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	Prove that every Borel set is Lebesgue measurable.	K1	CO1
2	17	State and prove the Lebesgue Dominated Convergence Theorem.	K2	CO2
3	18	State and prove Carathéodory's extension theorem regarding the construction of a countably additive measure from an outer measure.	K3	CO3
4	19	Prove that every convex function on an open interval is continuous.	K4	CO4
5	20	State and Prove the Radon-Nikodym Theorem.	K5	CO5

Z-Z-Z END