MSc DEGREE EXAMINATION MAY 2026
(Second Semester)

Branch – **MATHEMATICS**

MEASURE THEORY AND INTEGRATION

Time: Three Hours — Maximum: 75 Marks

SECTION-A (10 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 x 1 = 10)

1 A set E is said to be measurable if for each set A, $m^*A =$ ____.
(i) $m^*(A \cap E) + m^*(A \cap \overline{E})$ (ii) $m^*(A \cap E) - m^*(A \cap \overline{E})$
(iii) $m^*(A \cap E) \cdot m^*(A \cap \overline{E})$ (iv) $\dfrac{m^*(A \cap E)}{m^*(A \cap \overline{E})}$

2 If $m^*E = 0$, then E is ____.
(i) countable (ii) measurable
(iii) uncountable (iv) disjoint sets

3 A linear combination of characteristic function is known as_____.
(i) measurable function (ii) simple function
(iii) step function (iv) integrable function

4 If f is a nonnegative measurable function defined on a measurable set E, then $\int_E f =$ _____.
(i) $\inf_{h \le f} \int_E h$ (ii) $\inf_{h \le f_n} \int_E h$ (iii) $\sup_{h \le f_n} \int_E h$ (iv) $\sup_{h \le f} \int_E h$

5 If f is an integrable function on $[a,b]$, then indefinite integral of $F(x)$ on $[a,b]$ is ___________.
(i) $\int_a^x f(t)dt$ (ii) $\int f(t)dt$ (iii) $\int_a^x |f(t)|dt$ (iv) $\int |f(t)|dt$

6 If f is of bounded variation on $[a,b]$, then $T_a^b =$ _________.
(i) $P_a^b + N_a^b$ (ii) $P_a^b - N_a^b$ (iii) N_a^b (iv) P_a^b

7 A set E is said to be of _______, if E is the union of a countable collection of measurable sets of finite measure.
(i) finite measure (ii) σ-finite measure
(iii) semi-finite measure (iv) σ-algebra

8 Let E be a measurable set such that $0 < \nu E < \infty$. Then there is a positive set A contained in E with_____.
(i) $\nu A = 0$ (ii) $\nu A < 0$ (iii) $\nu A > 0$ (iv) $\nu A > 1$

9 If $A \in$ B, then $\mu^* A =$_______.
(i) A (ii) 1 (iii) 0 (iv) μA

10 If a set C in C is the union of a finite disjoint collection $\{C_i\}$ of sets in C, then $\mu C =$______.
(i) μ (ii) $\mu' C$ (iii) $\mu^* C$ (iv) $\sum \mu C_i$

Cont..

Answer **ALL** questions
ALL questions carry **EQUAL** Marks (5 x 5 = 25)

11 a Prove that the interval (a,∞) is measurable.

OR

b Define measurable function. (2)

c If f is a measurable function and $f = g$ almost everywhere, then prove that g is measurable. (3)

12 a If f and g are bounded measurable functions defined on a set E of finite measure, then prove that $\int_E (af + bg) = a\int_E f + b\int_E g$.

OR

b State and prove Fatou's lemma.

13 a Prove that a function f is of bounded variation on $[a,b]$ if and only if f is the difference of two monotone real-valued functions on $[a,b]$.

OR

b Prove that a function F is an indefinite integral if and only if it is absolutely continuous.

14 a If $E_i \in \mathrm{B}$, $\mu E_1 < \infty$ and $E_i \supset E_{i+1}$ then prove that $\mu\left(\bigcap_{i=1}^{\infty} E_i\right) = \lim_{n\to\infty} \mu E_n$.

OR

b State and prove Monotone convergence theorem.

15 a Prove that the set function μ^* is an outer measure.

OR

b Let x be a point of X and E a set in $R_{\sigma\delta}$. Then prove that E_x is a measurable subset of Y.

SECTION -C (40 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** Marks (5 x 8 = 40)

16 a Prove that the outer measure of an interval is its length.

OR

b State and prove the third principle of Littlewood's principles.

17 a State and prove bounded convergence theorem.

OR

b State and prove Lebesgue convergence theorem.

18 a State and prove Vitali lemma.

OR

b Let f be an integrable function on $[a,b]$ and suppose that $F(x) = F(a) + \int_a^x f(t)dt$, then prove that $F'(x) = f(x)$ for almost all x in $[a,b]$.

19 a State and prove Hahn decomposition theorem.

OR

b State and prove Radon – Nikodym theorem.

20 a State and prove Caratheodory theorem.

OR

b State and prove Fubini theorem.

Z-Z-Z END