PSG COLLEGE OF ARTS & SCIENCE

(AUTONOMOUS)

MSc DEGREE EXAMINATION MAY 2026

(First Semester)

Branch - **MATHEMATICS**

MATHEMATICAL STATISTICS

Time: Three Hours Maximum: 75 Marks

SECTION-A (10 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** marks $(10 \times 1 = 10)$

Module No.	Question No.	Question	K Level	CO
1	1	The expected value of a random variable X is a) A probability b) A constant c) A function d) Always zero	K1	CO1
	2	For a continuous random variable,P(X=a) is always a) 0 b) 1 c) Between 0 and 1 d) Undefined	K2	CO2
2	3	Expectation is a ______. a) random variable b) distribution c) probability d)constant	K1	CO1
	4	The second central moment of a random variable is equal to: a) Mean b) Variance c) Skewness d) Kurtosis	K2	CO1
3	5	The cumulant generating function (CGF) is defined as: a) $K_X(t) = M_X(t)$ b) $K_X(t) = E[X^t]$ c) $K_X(t) = logM_X(t)$ d) $K_X(t) = e^{M_X(t)}$	K1	CO1
	6	For a normal distribution $N(\mu, \sigma^2)$, the third cumulant is: a) 0 b) μ c) σ^2 d) $infinite$	K2	CO1
4	7	The mean and variance of a Poisson distribution are: a) Equal to λ b) Mean= λ,variance=λ^2 c) Mean=0,variance= λ d) Mean= λ,variance=$\sqrt{\lambda}$	K1	CO1
	8	The skewness of a Rectangular distribution is always (a) Positive (b) Negative (c) Zero (d) Infinite	K2	CO1
5	9	The Chi-square distribution is always (a) Symmetric b) Positively skewed (c) Negatively skewed (d) Normal for any n	K1	CO1
	10	Which distribution is heavier-tailed compared to the standard normal? (a) t-distribution (b) Chi-square distribution (c) F-distribution (d) Rectangular distribution	K2	CO2

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SECTION - B (35 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 × 7 = 35)

<table>
<tr><th>Module No.</th><th>Question No.</th><th>Question</th><th>K Level</th><th>CO</th></tr>
<tr><td rowspan="3">1</td><td>11.a.</td><td>The joint probability density function of two dimensional random variable X and Y is given by
$f(x,y) = 4x(1-y), 0 < x < 1, 0 < y < 1.$
Find marginal density functions of X and Y. Conditional density functions of X and Y. Also, find the probability of the event A, where $A: \{X, Y: (x,y): x > \frac{1}{2}, y < \frac{1}{2}\}$.</td><td rowspan="3">K2</td><td rowspan="3">CO2</td></tr>
<tr><td colspan="2">(OR)</td></tr>
<tr><td>11.b.</td><td>The probability distribution of a discrete random variable X is given by
<table>
<tr><td>X</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr>
<tr><td>P(X=x)</td><td>0.2</td><td>0.3</td><td>0.2</td><td>0.1</td><td>0.2</td></tr>
</table>
Find the probability distribution of $Y = k(x) = (x-1)^2$.</td></tr>
<tr><td rowspan="3">2</td><td>12.a.</td><td>The joint probability function of two dimensional random variables X and Y is given below
<table>
<tr><td></td><td>0</td><td>1</td><td>2</td><td>3</td><td>g(x)</td></tr>
<tr><td>0</td><td>2/8</td><td>1/8</td><td>1/8</td><td>1/8</td><td>5/8</td></tr>
<tr><td>1</td><td>1/8</td><td>1/8</td><td>1/8</td><td>-</td><td>3/8</td></tr>
<tr><td>h(y)</td><td>3/8</td><td>2/8</td><td>2/8</td><td>1/8</td><td>1.00</td></tr>
</table>
Find
a) V(X)
b) V(Y)
c) Cov(XY)</td><td rowspan="3">K3</td><td rowspan="3">CO3</td></tr>
<tr><td colspan="2">(OR)</td></tr>
<tr><td>12.b.</td><td>Let f(x) be the probability distribution function of the random variable X. Let us define a non-negative function $u(X)$ such that $E[u(X)]$ exists. Then prove for any constant C $P[u(x) \geq C] \leq \frac{E[u(X)]}{C}$.</td></tr>
<tr><td rowspan="3">3</td><td>13.a.</td><td>The joint probability density function of X and Y is given by
$f(x,y) = e^{-(x+y)}; x > 0, y > 0.$
Find cumulants of $U = X + Y$, and hence β_1 and β_2 of U.</td><td rowspan="3">K3</td><td rowspan="3">CO3</td></tr>
<tr><td colspan="2">(OR)</td></tr>
<tr><td>13.b.</td><td>The joint probability density function of two dimensional random variables x_1 and x_2 is given by
$f(x_1, x_2) = e^{-(x_1+x_2)}; x_1 > 0, x_2 > 0.$
Find characteristic function of the random variables and hence moments.</td></tr>
<tr><td rowspan="3">4</td><td>14.a.</td><td>Derive the moment generating functions of the Poisson distribution.</td><td rowspan="3">K3</td><td rowspan="3">CO3</td></tr>
<tr><td colspan="2">(OR)</td></tr>
<tr><td>14.b.</td><td>Find mean deviation about mean of normal distribution.</td></tr>
<tr><td rowspan="3">5</td><td>15.a.</td><td>Prove that normal distribution is a particular case of χ^2-distribution with $n-1$ degrees of freedom.</td><td rowspan="3">K3</td><td rowspan="3">CO3</td></tr>
<tr><td colspan="2">(OR)</td></tr>
<tr><td>15.b.</td><td>If X_1 and X_2 are two independent random variables having common density function $f(x) = e^{-x}, 0 < x < \infty$, show that $u = {x_1}/{x_2}$ has an F-distribituion.</td></tr>
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SECTION -C (30 Marks)

Answer **ANY THREE** questions

ALL questions carry **EQUAL** Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	The two dimensional continuous random variables X and Y have the joint probability density function $f(x,y) = \begin{cases} \frac{3}{2}y^2, 0 \le x \le 2, 0 \le y \le 1 \\ 0, otherwise \end{cases}$ Find conditional probability functions of both variables. Are the variables independent? Find probability of the event A, where $A: \{X: \frac{1}{2} \le x \le 1\}$.	K4	CO4
2	17	The joint probability density function of two dimensional random variables X and Y is given by $f(x,y) = 21x^2y^3, 0 < x < y < 1$. Find a) V(X-2Y) b) V(X/Y) c) V(X)	K4	CO4
3	18	The continuous random variable X has the probability density function $f(x) = \theta e^{\theta x}; x > 0, \theta > 0$. Find moments of the distribution and comment on its shape characteristics.	K4	CO4
4	19	Find moment generating functions of the Binomial distribution.	K4	CO4
5	20	Let $x_1, x_2, \dots, x_n$ be a random sample from normal population $N(\mu, \sigma^2)$. Then prove (i) $\bar{x}$ and (ii) $\frac{ns^2}{\sigma^2}$ are independently distributed as normal and chi-square respectively.	K5	CO5

Z-Z-Z END