(AUTONOMOUS)

MSc DEGREE EXAMINATION MAY 2026

(Fourth Semester)

Branch - **MATHEMATICS**

MATHEMATICAL METHODS

Time: Three Hours Maximum: 75 Marks

SECTION-A (10 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** marks (10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	Which of the following is of Fredholm integral equation of the first kind? a) $h(s)g(s) = f(s) + \lambda \int_a K(s,t)g(t)dt$ b) $f(s) + \lambda \int_a^b K(s,t)g(t)dt = 0$ c) $g(s) = f(s) + \lambda \int_a^b K(s,t)g(t)dt$ d) $g(s) = \lambda \int_a^b K(s,t)g(t)dt$.	K1	CO1
	2	When one or both limits of integration become infinite or when the kernel becomes infinite at one or more points within the range of integration, then it is called ____ integral equations. a) Fredholm b) Volterra c) singular d) none of these.	K2	CO2
2	3	In $g(s) = f(s) + \lambda \int_a^s \Gamma(s,t;\lambda)g(t)dt$, the function $\Gamma(s,t;\lambda)$ is known as ____. a) Gamma kernel b) generating factor c) integral curve d) resolvent kernel.	K1	CO1
	4	The determinant formed by the values of the kernel at all points is called ____ determinant. a) resolvent b) Lovitt c) Hilbert d) Fredholm.	K2	CO1
3	5	The initial value problems for ordinary differential equations can be converted to integral equations of _____ type. a) Fredholm b) Volterra c) Abel d) homogeneous.	K1	CO1
	6	Which of the following is a singular integral equation? a) Fredholm b) Volterra c) Abel d) homogeneous.	K2	CO2
4	7	The curves $y = y(x)$ and $y = y_1(x)$ are close in the sense of _____ proximity if the absolute value of the difference $y(x) - y_1(x)$ is small. a) zero order b) first order c) kth order d) none of these.	K1	CO1
	8	The Euler's equation for a functional of the form $\int_{x_0}^{x_1} F(x,y,y')\,dx$ is _____. a) $F_y = 0$ b) $F_{y'} = 0$ c) $F - y'F_{y'} = 0$ d) $F_{y'y'} = 0$.	K2	CO2
5	9	$\left(F_{yy} - \frac{d}{dx} F_{yy'}\right)u - \frac{d}{dx}\left(F_{y'y'}u'\right) = 0$ is called ____ equation. a) Euler b) Jacobi c) Euler- Jacobi d) Hamilton-Jacobi.	K1	CO1
	10	The condition $F_{y'y'} > 0$ or $F_{y'y'} < 0$ is due to ____. a) Jacobi b) Hamilton c) Weierstrass d) Legendre.	K2	CO1

Cont...

SECTION – B (35 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	Write a short note on special kinds of kernels.	K4	CO3
		(OR)		
	11.b.	Solve the Fredholm integral equation of the second kind $g(s) = s + \lambda \int_0^1 (st^2 + s^2 t) g(t)dt$.		
2	12.a.	Find the Neumann series for the solution of the integral equation $g(s) = (1 + s) + \lambda \int_0^s (s - t)\, g(t)dt$.	K4	CO3
		(OR)		
	12.b.	Solve the integral equation $g(s) = 1 + \lambda \int_0^\pi [\sin(s + t)] g(t)dt$.		
3	13.a.	Reduce the initial value problem $y''(s) + \lambda y(s) = F(s)$, $y(0) = 1$, $y'(0) = 0$ to a Volterra integral equation.	K5	CO4
		(OR)		
	13.b.	Solve the integral equation $f(s) = \int_a^s \frac{g(t)dt}{(s^2 - t^2)^\alpha}$, $0 < \alpha < 1;\ a < s < b$.		
4	14.a.	Determine the curves on which the functional $v[y(x)] = \int_0^1 (y'^2 + 12xy)dx$ can be extremized, $y(0) = 0$, $y(1) = 1$.	K3	CO3
		(OR)		
	14.b.	Find the extremals of the functional $v[y(x), z(x)] = \int_0^{\frac{\pi}{2}} (y'^2 + z'^2 + 2yz)dx$, $y(0) = 0$, $y\left(\frac{\pi}{2}\right) = 1$, $z(0) = 0$, $z\left(\frac{\pi}{2}\right) = -1$.		
5	15.a.	Explain the two conditions for a functional v to achieve an extremum on the curve C.	K3	CO3
		(OR)		
	15.b.	Is the Jacobi condition fulfilled for the extremal of the functional $v = \int_0^a (y'^2 - y^2)\, dx$ that passes through the points A (0, 0) and B (a, 0)?		

SECTION -C (30 Marks)

Answer **ANY THREE** questions

ALL questions carry **EQUAL** Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	Find the resolvent kernel for the integral equation $g(s) = f(s) + \lambda \int_{-1}^1 (st + s^2 t^2) g(t)dt$.	K4	CO3
2	17	Solve the integral equation $g(s) = s + \lambda \int_0^1 \left[st + (st)^{1/2}\right] g(t)dt$.	K4	CO3
3	18	Reduce the boundary value problem $y''(s) + A(s)y'(s) + B(s)y = F(s)$, $y(a) = y_0$, $y(b) = y_1$ to Fredholm integral equation.	K5	CO4
4	19	State and solve the minimum surface of revolution problem.	K4	CO3
5	20	Test for an extremum of the functional $v[y(x)] = \frac{1}{\sqrt{2gy}} \int_0^{x_1} \frac{\sqrt{1+y'^2}}{\sqrt{y}} dx$; $y(0) = 0$, $y(x_1) = y_1$.	K5	CO4

Z-Z-Z END