(Second Semester)

Branch - **MATHEMATICS**

COMPLEX ANALYSIS

Time: Three Hours | Maximum: 75 Marks

SECTION-A (10 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** marks (10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	Choose the transformation under which the cross ratio of four complex numbers is invariant: (i) Translation (ii) Rotation (iii) Linear fractional transformation (iv) Conjugation	K1	CO1
	2	Identify the condition under which a function is analytic: (i) Laplace equation (ii) Cauchy-Riemann equations (iii) Taylor theorem (iv) Mean value theorem	K2	CO1
2	3	Choose what the argument principle relates: (i) Zeros and poles (ii) Derivatives (iii) Integrals and limits (iv) Functions and series	K1	CO2
	4	Select the equation satisfied by a harmonic function: (i) Cauchy equation (ii) Laplace equation (iii) Euler equation (iv) Wave equation	K2	CO2
3	5	Choose the type of functions for which a Taylor series is valid: (i) Non-analytic functions (ii) Periodic functions (iii) Discontinuous functions (iv) Analytic functions	K1	CO3
	6	Identify where Jensen's formula is applied: (i) Harmonic functions (ii) Entire functions (iii) Residue theory (iv) Integration	K2	CO3
4	7	Choose what a conformal mapping preserves: (i) Length (ii) Area (iii) Angles (iv) Volume	K1	CO4
	8	Select the concept to which Harnack's principle relates: (i) Conformal maps (ii) Residues (iii) Poles (iv) Harmonic functions	K2	CO4
5	9	Choose the condition satisfied by a periodic function: (i) $f(z+p)=f(z)$ (ii) $f(z)=0$ (iii) $f'(z)=f(z)$ (iv) None	K1	CO5
	10	Identify the type of function represented by the Weierstrass $\wp$-function: (i) Rational (ii) Elliptic (iii) Polynomial (iv) Harmonic	K2	CO5

Cont...

ALL questions carry EQUAL Marks (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	Explain the statement and proof for the necessary and sufficient condition under which a line integral depends only on the end points.	K2	CO1
		(OR)		
	11.b.	Explain the statement and proof for the Cauchy's integral formula.		
2	12.a.	Construct the statement and proof of the argument principle.	K3	CO2
		(OR)		
	12.b.	Construct the statement and proof of the Mean Value Property for Harmonic Functions.		
3	13.a.	Discover the statement and proof of the Hurwitz's Theorem.	K4	CO3
		(OR)		
	13.b.	Examine that the following representation is valid in the largest open disk of center z_0 contained in Ω where f(z) is analytic in the region Ω, containing z_0. $f(z) = f(z_0) + \frac{f'(z_0)}{1!}(z - z_0) + \ldots + \frac{f^{(n)}(z_0)}{n!}(z - z_0)^n + \ldots$		
4	14.a.	State and prove Harnack's Principle.	K5	CO4
		(OR)		
	14.b.	Justify that a continuous function u(z) which satisfies the mean - value property is necessarily harmonic.		
5	15.a.	Prove that a non constant elliptic function has equally many poles as it has zeros.	K5	CO5
		(OR)		
	15.b.	Justify that an elliptic function without poles is a constant.		

SECTION -C (30 Marks)

Answer **ANY THREE** questions

ALL questions carry **EQUAL** Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	Explain the statement and proof of the Cauchy's theorem for a rectangle.	K2	CO1
2	17	Construct the statement and proof for the Schwarz' theorem.	K3	CO2
3	18	Discover the statement and proof for the Mittag Leffler Theorem.	K4	CO3
4	19	State and prove Schwartz Christoeffel formula.	K5	CO4
5	20	Justify that a discrete module consists either of zero alone, of the integral multiples $n\omega$ of a single complex number $\omega \neq 0$, or of all linear combinations $n_1\omega_1 + n_2\omega_2$ with integral coefficients of two numbers ω_1, ω_2 with nonreal ratio ω_2/ω_1.	K5	CO5

Z-Z-Z END